Chapter -01-Units, Physical Quantities, and Vectors

General Physics I / PHYS 140

College of Science

Department of Physics

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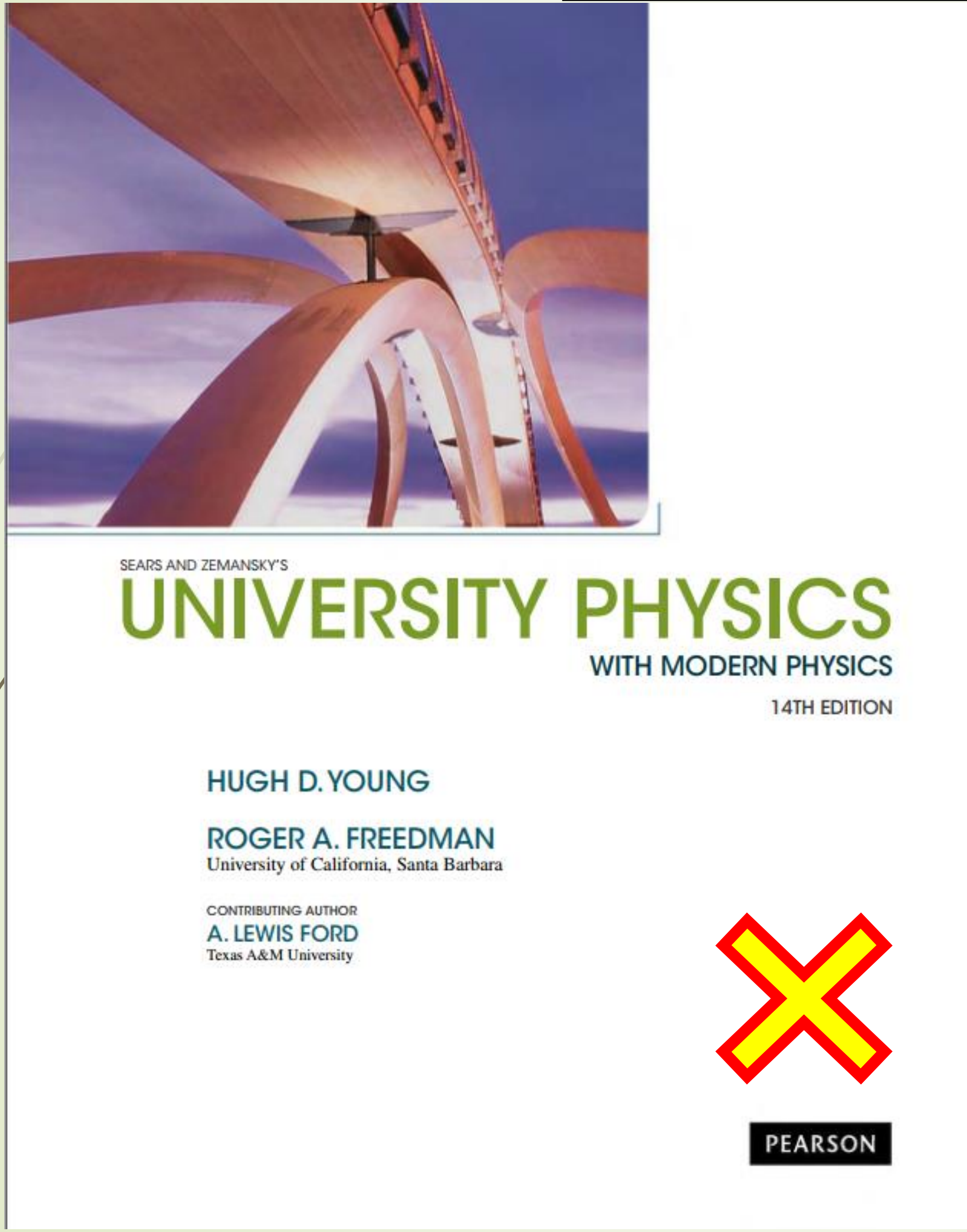
Course Title:	**General Physics 1, Phys. 140**
Course Code:	**0814-140**
Program:	**Bachelor of Engineering**
Department:	**Physics**
College:	**College of Science**
Institution:	**King Faisal University**

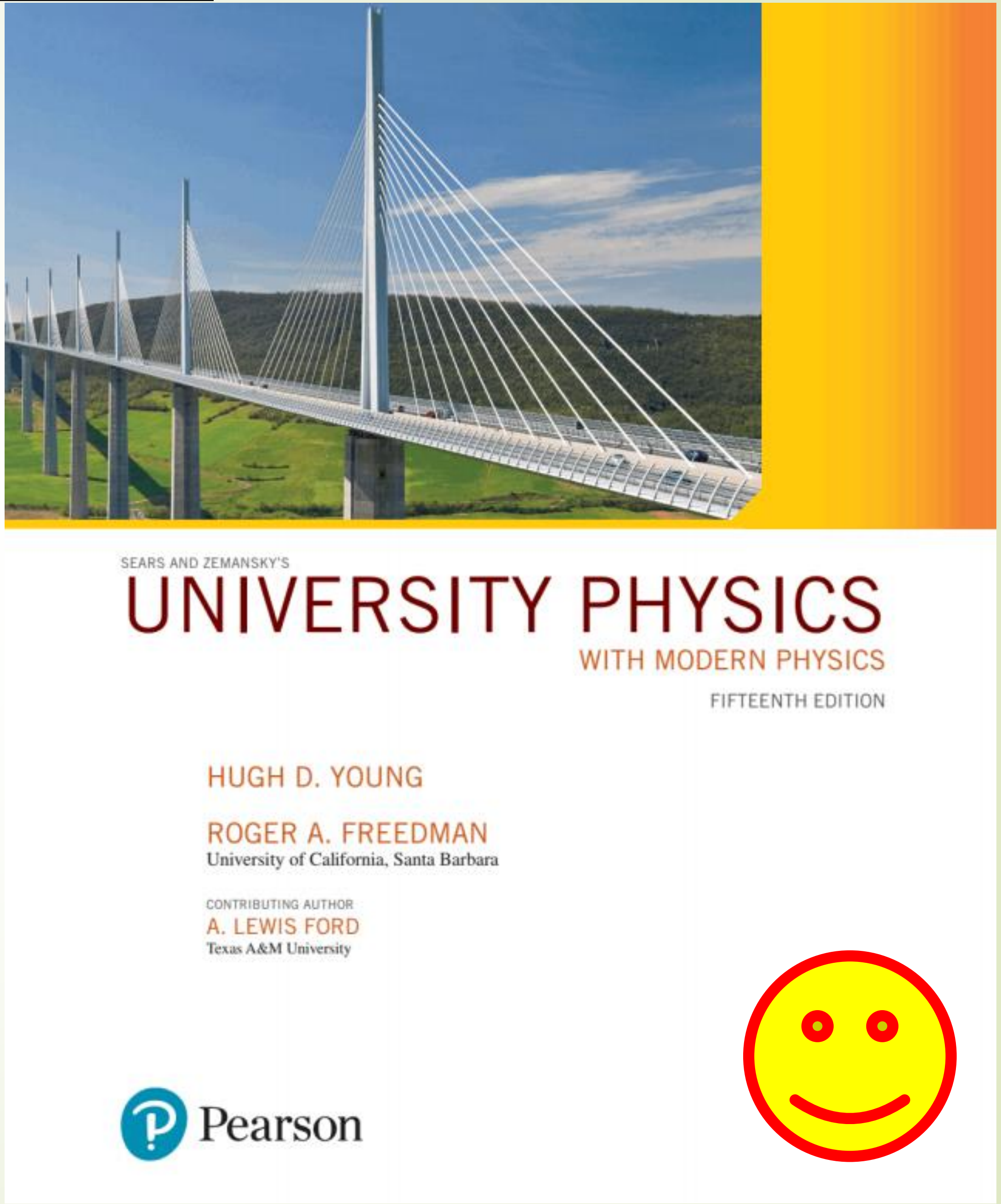
No	Mode of Instruction	Percentage
1	Traditional classroom	100%

Outline

No	List of Topics	Contact Hours
1	Units, Physical Quantities and Vectors	6
2	Motion Along A Straight Line (ID-Motion)	4.5
3	Motion in 2 or 3 Dimensions (2D/3D Motion)	4.5
4	Newtons Laws of Motion	2
5	Applying Newtons Laws	4
6	Work And Kinetic Energy	4.5
7	Potential Energy and Energy Conservation	4.5
8	Momentum, Impulse, And Collisions	6
9	Rotation of Rigid Bodies	6
10	Dynamics of Rotational Motion	6
Total		45

Reference book





Grading

Assessment	Task	Week due	Percentage
1	Quizzes (4)	3, 5, 7, 9	20%
2	Mid Term Exam 1	6	40%
3	Final Exam	11 or 12	40%
	Total		100%

Grading Scale in %:

95-100	A+
90-94	A
85-89	B+
80-84	B
75-79	C+
70-74	C
65-69	D +
60-64 D	
Less than	60 F

Evaluation System

Evaluation Areas/Issues	Evaluators	Evaluation Methods
Effectiveness of Teaching	Students	Indirect (Online survey conducted at the end of the course).
Effectiveness of Teaching	Faculty members	Direct (Classroom observation using the QMS teaching Observation Concepts and Teaching Observation Proforma (QMS Annex O and Annex P)).
Assessment of Faculty Members	Program Leader	Direct/Indirect (Performance Assessment of Faculty Members using the QMS Annex N form)

1.1 THE NATURE OF PHYSICS

Physics is an experimental science.

Ask questions → Design experiments ⇄ Make observations → Find pattern (Theory)

- ❑ **Theories can be revised by new observation.**
- ❑ **Theories have a range of validity.**
- ❑ **Measurements and numbers are required to perform experiments and explain the results.**

Problem-Solving Strategies;

Problem-Solving Strategies that offer techniques for setting up and solving problems efficiently and accurately.

ISEE

- **Identify** → **Determine target variable and given quantities.**
- **Setup** → **Choose equations based on the known and unknown values.**
- **Execute** → **Do the math.**
- **Evaluate** → **Dose answer make the sense.**

1.3 STANDARDS AND UNITS

In mechanics, three fundamental quantities are used;

Time (second) s	Length (meter) m	Mass (Kilogram) Kg

Other quantities in mechanics can be expressed in terms of the three fundamental quantities and are called derived quantities.

Area: A product of two lengths.

Speed: A ratio of a length to a time interval.

Density: A ratio of mass to volume.

TABLE 1.1 Some Units of Length, Mass, and Time

Length	Mass	Time
1 nanometer = 1 nm = 10^{-9} m (a few times the size of the largest atom)	1 microgram = 1 μg = 10^{-6} g = 10^{-9} kg (mass of a very small dust particle)	1 nanosecond = 1 ns = 10^{-9} s (time for light to travel 0.3 m)
1 micrometer = 1 μm = 10^{-6} m (size of some bacteria and other cells)	1 milligram = 1 mg = 10^{-3} g = 10^{-6} kg (mass of a grain of salt)	1 microsecond = 1 μs = 10^{-6} s (time for space station to move 8 mm)
1 millimeter = 1 mm = 10^{-3} m (diameter of the point of a ballpoint pen)	1 gram = 1 g = 10^{-3} kg (mass of a paper clip)	1 millisecond = 1 ms = 10^{-3} s (time for a car moving at freeway speed to travel 3 cm)
1 centimeter = 1 cm = 10^{-2} m (diameter of your little finger)		
1 kilometer = 1 km = 10^{3} m (distance in a 10-minute walk)		

(a) 10^{26} m Limit of the observable universe

(b) 10^{11} m Distance to the sun

(c) 10^{7} m Diameter of the earth

(d) 1 m Human dimension

(e) 10^{-5} m Diameter of a red blood cell

(f) 10^{-10} m Radius of an atom

(g) 10^{-14} m Radius of an atomic nucleus

From very big to very small

1.4 USING AND CONVERTING UNITS

An equation must always be dimensionally consistent. You can't add apples and automobiles; two terms may be added or equated only if they have the same units.

For example, if a body moving with constant speed *v* travels a distance *d* in a time *t*, these quantities are related by the equation;

For example the equation distance= speed x time

$$d\,(\mathrm{m}) = v\,(\mathrm{m/s}) \,.\, t\,(\mathrm{s})$$

Both sides must be in the same units and dimensions, if *d* is in meters, *vt* also must be in meters.

$$10\ \mathrm{m} = \left(2\,\frac{\mathrm{m}}{\cancel{\mathrm{s}}}\right)(5\ \cancel{\mathrm{s}})$$

EXAMPLE 1.1 CONVERTING SPEED UNITS

SOLUTION

The world land speed record of 763.0 mi/h was set on October 15, 1997, by Andy Green in the jet-engine car *Thrust SSC*. Express this speed in meters per second.

SOLUTION

IDENTIFY, SET UP, and EXECUTE: We need to convert the units of a speed from mi/h to m/s. We must therefore find unit multipliers that relate (i) miles to meters and (ii) hours to seconds. In Appendix E we find the equalities 1 mi = 1.609 km, 1 km = 1000 m, and 1 h = 3600 s. We set up the conversion as follows, which ensures that all the desired cancellations by division take place:

$$763.0\ \text{mi/h} = \left(763.0\ \frac{\cancel{\text{mi}}}{\cancel{\text{h}}}\right)\left(\frac{1.609\ \cancel{\text{km}}}{1\ \cancel{\text{mi}}}\right)\left(\frac{1000\ \text{m}}{1\ \cancel{\text{km}}}\right)\left(\frac{1\ \cancel{\text{h}}}{3600\ \text{s}}\right)$$
$$= 341.0\ \text{m/s}$$

EVALUATE: This example shows a useful rule of thumb: A speed expressed in m/s is a bit less than half the value expressed in mi/h, and a bit less than one-third the value expressed in km/h. For example, a normal freeway speed is about 30 m/s = 67 mi/h = 108 km/h, and a typical walking speed is about 1.4 m/s = 3.1 mi/h = 5.0 km/h.

EXAMPLE 1.2 CONVERTING VOLUME UNITS

One of the world's largest cut diamonds is the First Star of Africa (mounted in the British Royal Sceptre and kept in the Tower of London). Its volume is 1.84 cubic inches. What is its volume in cubic centimeters? In cubic meters?

SOLUTION

IDENTIFY, SET UP, and EXECUTE: Here we are to convert the units of a volume from cubic inches (in.^3) to both cubic centimeters (cm^3) and cubic meters (m^3). Appendix E gives us the equality $1 \text{ in.} = 2.540 \text{ cm}$, from which we obtain $1 \text{ in.}^3 = (2.54 \text{ cm})^3$. We then have

$$1.84 \text{ in.}^3 = (1.84 \text{ in.}^3)\left(\frac{2.54 \text{ cm}}{1 \text{ in.}}\right)^3$$

$$= (1.84)(2.54)^3 \frac{\cancel{\text{in.}^3}\ \text{cm}^3}{\cancel{\text{in.}^3}} = 30.2 \text{ cm}^3$$

Appendix E also gives us $1 \text{ m} = 100 \text{ cm}$, so

$$30.2 \text{ cm}^3 = (30.2 \text{ cm}^3)\left(\frac{1 \text{ m}}{100 \text{ cm}}\right)^3$$

$$= (30.2)\left(\frac{1}{100}\right)^3 \frac{\cancel{\text{cm}^3}\ \text{m}^3}{\cancel{\text{cm}^3}} = 30.2 \times 10^{-6} \text{ m}^3$$

$$= 3.02 \times 10^{-5} \text{ m}^3$$

EVALUATE: Following the pattern of these conversions, can you show that $1 \text{ in.}^3 \approx 16 \text{ cm}^3$ and that $1 \text{ m}^3 \approx 60{,}000 \text{ in.}^3$?

1.5 UNCERTAINTY AND SIGNIFICANT FIGURES

Measurements always have uncertainties;

The uncertainties depends on the measuring devices.

For example meter stick and micrometer caliper.

Example, the thickness of your mobile phone is announced to be 5.15 mm

- **With ordinary ruler, precise to 1mm.**

[you can write 5 mm not 5.0, 5.00, 5.10 or 5.15]

- **With a micrometer, precise to 0.01 mm.**

[you can write 5.15 mm]

- **Smaller uncertainty is a more accurate measurement.**

The uncertainty in the measurement is indicated by the number of <u>significant figures</u>.

<u>Significant Figures;</u> The digit that are known with the certainty plus the one digit.

TABLE 1.2 Using Significant Figures

Multiplication or division:
Result can have no more significant figures than the factor with the fewest significant figures:

$$\frac{0.745 \times 2.2}{3.885} = 0.42$$

$$1.32578 \times 10^{7} \times 4.11 \times 10^{-3} = 5.45 \times 10^{4}$$

Addition or subtraction:
Number of significant figures is determined by the term with the largest uncertainty (i.e., fewest digits to the right of the decimal point):

$$27.153 + 138.2 - 11.74 = 153.6$$

EXAMPLE 1.3 SIGNIFICANT FIGURES IN MULTIPLICATION

The rest energy E of an object with rest mass m is given by Albert Einstein's famous equation $E = mc^2$, where c is the speed of light in vacuum. Find E for an electron for which (to three significant figures) $m = 9.11 \times 10^{-31}$ kg. The SI unit for E is the joule (J); $1\ \text{J} = 1\ \text{kg} \cdot \text{m}^2/\text{s}^2$.

SOLUTION

IDENTIFY and SET UP: Our target variable is the energy E. We are given the value of the mass m; from Section 1.3 (or Appendix F) the speed of light is $c = 2.99792458 \times 10^8$ m/s.

EXECUTE: Substituting the values of m and c into Einstein's equation, we find

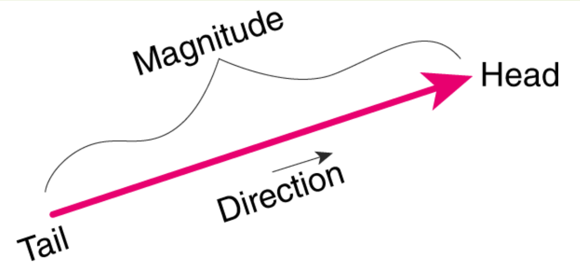
$$\begin{aligned} E &= (9.11 \times 10^{-31}\ \text{kg})(2.99792458 \times 10^8\ \text{m/s})^2 \\ &= (9.11)(2.99792458)^2(10^{-31})(10^8)^2\ \text{kg} \cdot \text{m}^2/\text{s}^2 \\ &= (81.87659678)(10^{[-31+(2\times 8)]})\ \text{kg} \cdot \text{m}^2/\text{s}^2 \\ &= 8.187659678 \times 10^{-14}\ \text{kg} \cdot \text{m}^2/\text{s}^2 \end{aligned}$$

Since the value of m was given to only three significant figures, we must round this to

$$E = 8.19 \times 10^{-14}\ \text{kg} \cdot \text{m}^2/\text{s}^2 = 8.19 \times 10^{-14}\ \text{J}$$

EVALUATE: While the rest energy contained in an electron may seem ridiculously small, on the atomic scale it is tremendous. Compare our answer to 10^{-19} J, the energy gained or lost by a single atom during a typical chemical reaction. The rest energy of an electron is about 1,000,000 times larger! (We'll discuss the significance of rest energy in Chapter 37.)

1.7 VECTORS AND VECTOR ADDITION



Vectors → **Vectors have both magnitude and direction.**

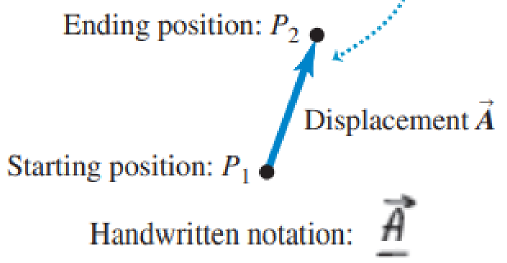
Displacement – area – velocity – Force – torque, etc.

Scalars → **Scalars have magnitude only.**

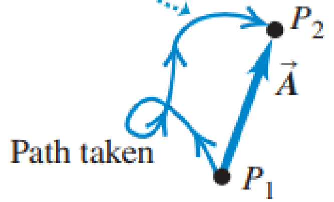
Mass –time –speed –distance - temperature – density – energy – etc.

1.9 Displacement as a vector quantity.

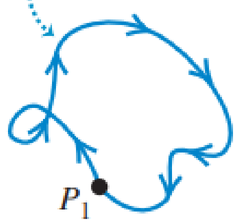
(a) We represent a displacement by an arrow that points in the direction of displacement.



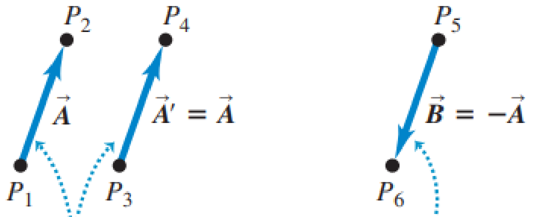
(b) A displacement is always a straight arrow directed from the starting position to the ending position. It does not depend on the path taken, even if the path is curved.



(c) Total displacement for a round trip is 0, regardless of the path taken or distance traveled.



1.10 The meaning of vectors that have the same magnitude and the same or opposite direction.



Displacements $\vec{A}$ and $\vec{A}'$ are equal because they have the same length and direction.

Displacement $\vec{B}$ has the same magnitude as $\vec{A}$ but opposite direction; $\vec{B}$ is the negative of $\vec{A}$.

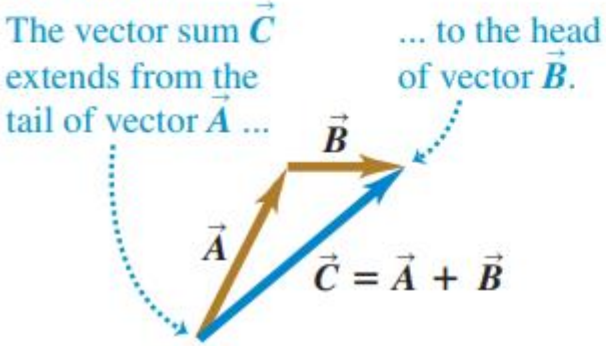
Equal and opposite vectors

Vector Addition and Subtraction

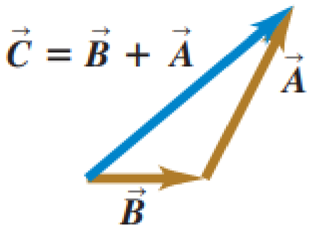
Because vectors have different orientation in space, we can not add them ~~algebraically~~. Instead, we use the vector addition that depends on trigonometry.

1.11 Three ways to add two vectors.

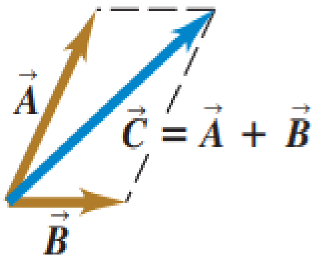
(a) We can add two vectors by placing them head to tail.



(b) Adding them in reverse order gives the same result: $\vec{A} + \vec{B} = \vec{B} + \vec{A}$. The order doesn't matter in vector addition.

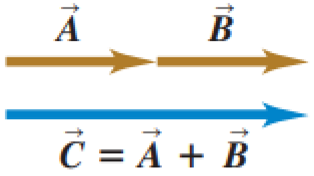


(c) We can also add two vectors by placing them tail to tail and constructing a parallelogram.

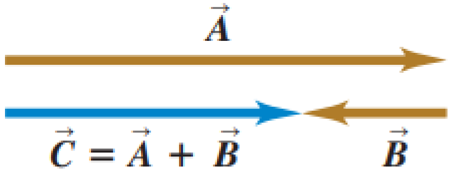


1.12 Adding vectors that are (a) parallel and (b) antiparallel.

(a) Only when vectors $\vec{A}$ and $\vec{B}$ are parallel does the magnitude of their vector sum $\vec{C}$ equal the sum of their magnitudes: $C = A + B$.



(b) When $\vec{A}$ and $\vec{B}$ are antiparallel, the magnitude of their vector sum $\vec{C}$ equals the *difference* of their magnitudes: $C = |A - B|$.



Commutative properties of the vector addition;

1.13 Several constructions for finding the vector sum $\vec{A} + \vec{B} + \vec{C}$.

(a) To find the sum of these three vectors ...

(b) ... add $\vec{A}$ and $\vec{B}$ to get $\vec{D}$ and then add $\vec{C}$ to $\vec{D}$ to get the final sum (resultant) $\vec{R}$ ...

(c) ... or add $\vec{B}$ and $\vec{C}$ to get $\vec{E}$ and then add $\vec{A}$ to $\vec{E}$ to get $\vec{R}$ ...

(d) ... or add $\vec{A}$, $\vec{B}$, and $\vec{C}$ to get $\vec{R}$ directly ...

(e) ... or add $\vec{A}$, $\vec{B}$, and $\vec{C}$ in any other order and still get $\vec{R}$.

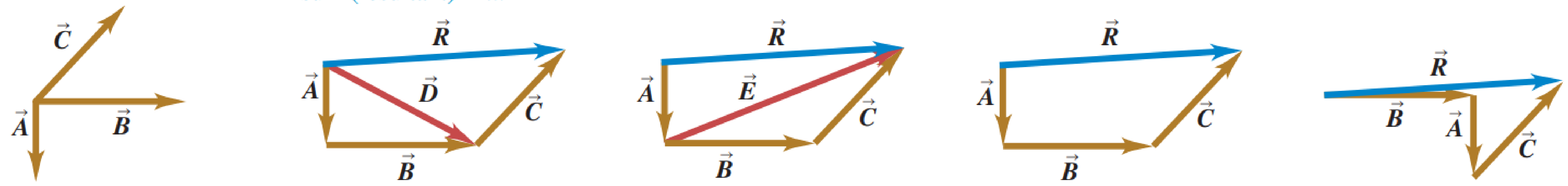


Figure 1.13a shows *three* vectors $\vec{A}$, $\vec{B}$, and $\vec{C}$. To find the vector sum of all three, in Fig. 1.13b we first add $\vec{A}$ and $\vec{B}$ to give a vector sum $\vec{D}$; we then add vectors $\vec{C}$ and $\vec{D}$ by the same process to obtain the vector sum $\vec{R}$:

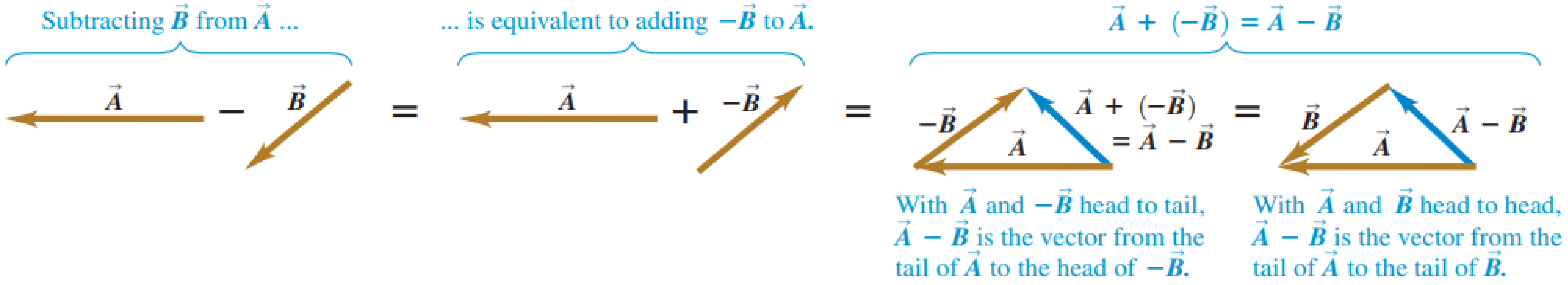
$$\vec{R} = (\vec{A} + \vec{B}) + \vec{C} = \vec{D} + \vec{C}$$

Alternatively, we can first add $\vec{B}$ and $\vec{C}$ to obtain vector $\vec{E}$ (Fig. 1.13c), and then add $\vec{A}$ and $\vec{E}$ to obtain $\vec{R}$:

$$\vec{R} = \vec{A} + (\vec{B} + \vec{C}) = \vec{A} + \vec{E}$$

Subtracting vectors;

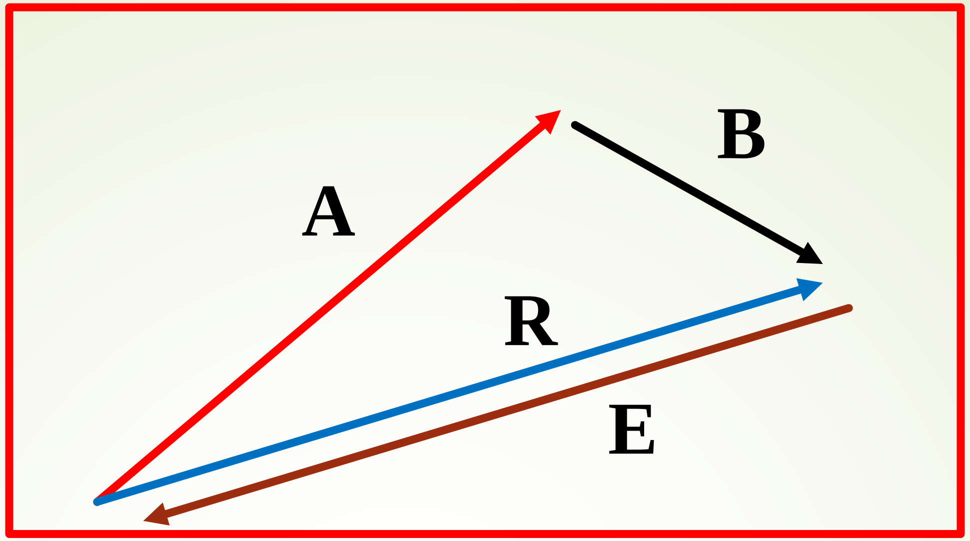
1.14 To construct the vector difference $\vec{A} - \vec{B}$, you can either place the tail of $-\vec{B}$ at the head of $\vec{A}$ or place the two vectors $\vec{A}$ and $\vec{B}$ head to head.



We can *subtract* vectors as well as add them. To see how, recall that vector $-\vec{A}$ has the same magnitude as $\vec{A}$ but the opposite direction. We define the difference $\vec{A} - \vec{B}$ of two vectors $\vec{A}$ and $\vec{B}$ to be the vector sum of $\vec{A}$ and $-\vec{B}$:

$$\vec{A} - \vec{B} = \vec{A} + (-\vec{B}) \tag{1.4}$$

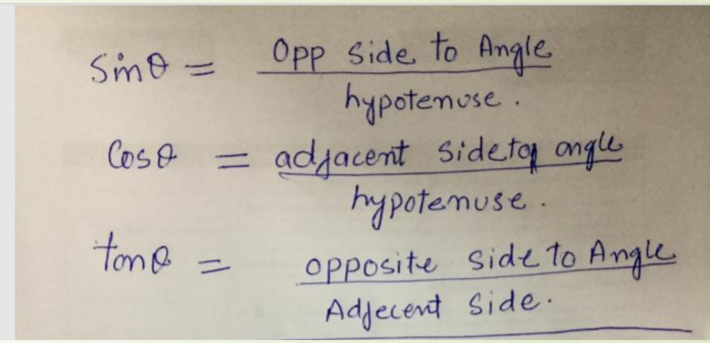
Resultant and Equilibrant;



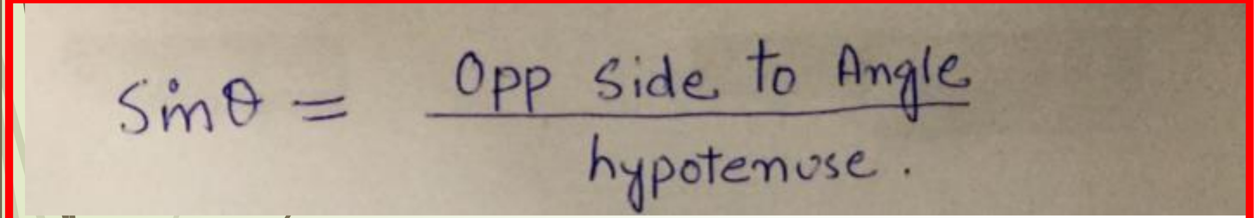
R = Resultant Vector;

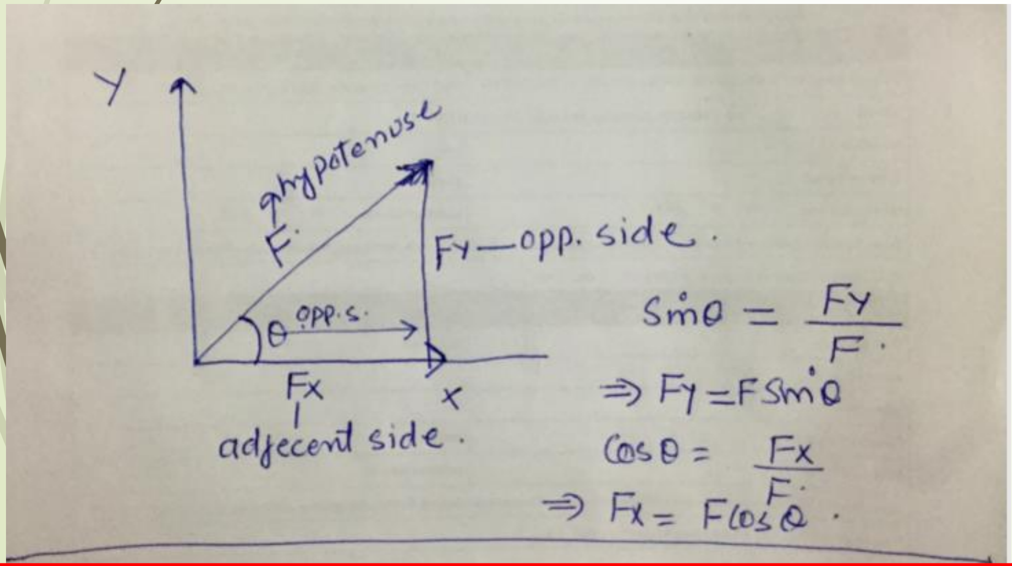
E = Equilibrant Vector;

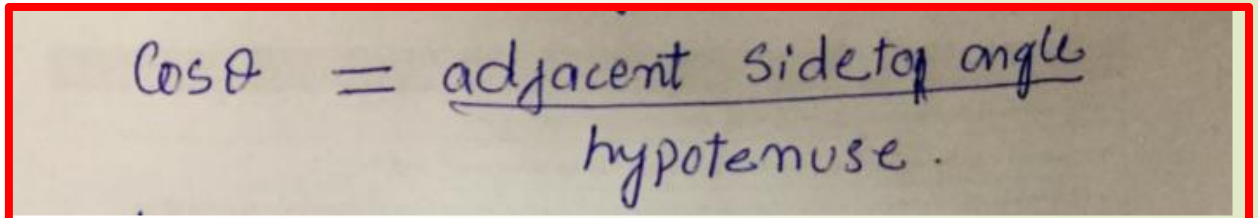
E = -R

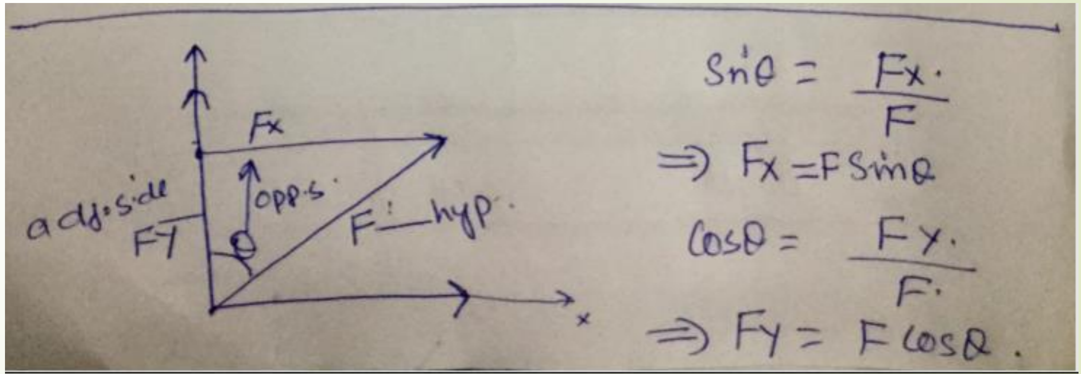


Pythagorean theorem, OR Pythagoras's theorem;





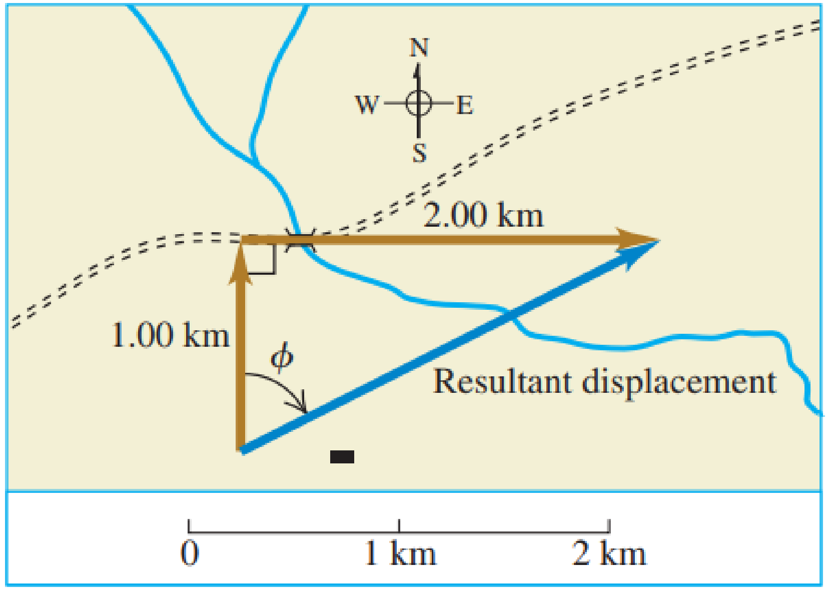




EXAMPLE 1.5 ADDING TWO VECTORS AT RIGHT ANGLES

A cross-country skier skis 1.00 km north and then 2.00 km east on a horizontal snowfield. How far and in what direction is she from the starting point?

1.16 The vector diagram, drawn to scale, for a ski trip.



SOLUTION

IDENTIFY and SET UP: The problem involves combining two displacements at right angles to each other. This vector addition amounts to solving a right triangle, so we can use the Pythagorean theorem and simple trigonometry. The target variables are the skier's straight-line distance and direction from her starting point. **Figure 1.16** is a scale diagram of the two displacements and the resultant net displacement. We denote the direction from the starting point by the angle ϕ (the Greek letter phi). The displacement appears to be a bit more than 2 km. Measuring the angle with a protractor indicates that ϕ is abo

EXECUTE: The distance from the starting point to the ending point is equal to the length of the hypotenuse:

$$\sqrt{(1.00\text{ km})^2 + (2.00\text{ km})^2} = 2.24\text{ km}$$

A little trigonometry (from Appendix B) allows us to find angle ϕ:

$$\tan\phi = \frac{\text{Opposite side}}{\text{Adjacent side}} = \frac{2.00\text{ km}}{1.00\text{ km}} = 2.00$$

$$\phi = \arctan 2.00 = 63.4^\circ$$

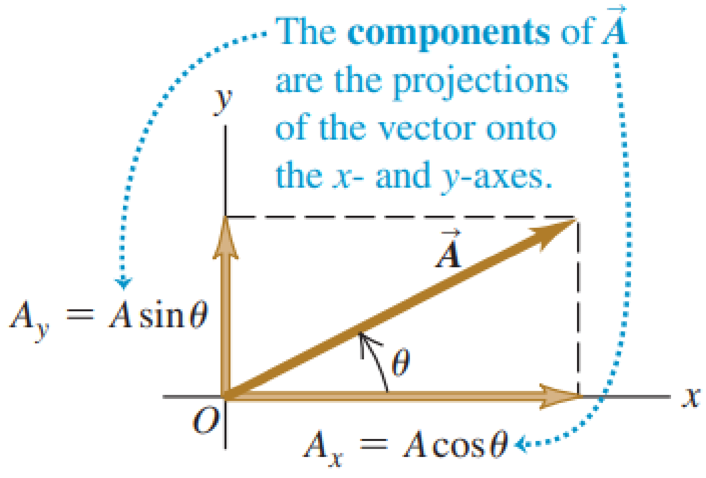
We can describe the direction as 63.4° east of north or $90^\circ - 63.4^\circ = 26.6^\circ$ north of east.

EVALUATE: Our answers (2.24 km and $\phi = 63.4^\circ$) are close to our predictions. In Section 1.8 we'll learn how to easily add two vectors *not* at right angles to each other.

1.8 COMPONENTS OF VECTORS

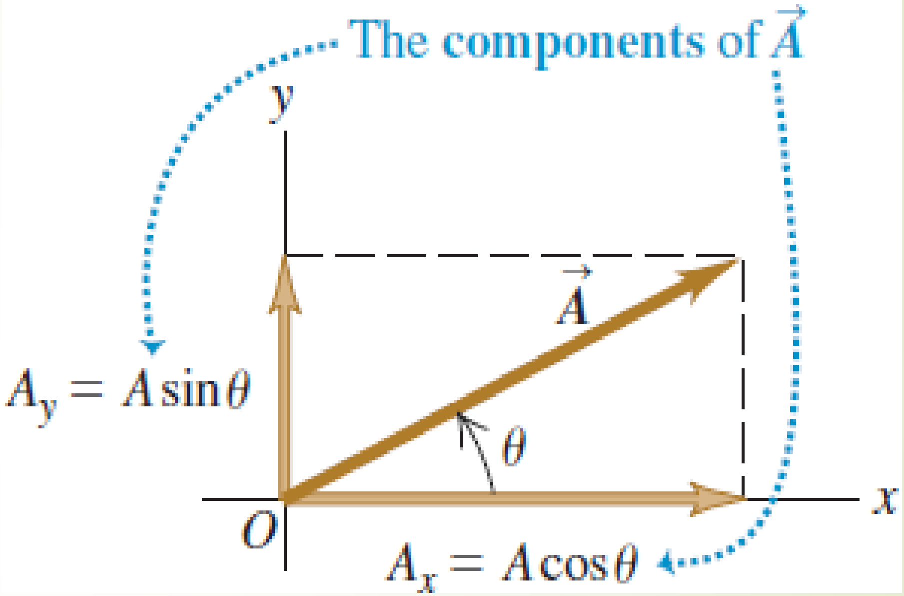
$$\vec{A} = \vec{A}_x + \vec{A}_y$$

1.17 Representing a vector $\vec{A}$ in terms of its components A_x and A_y.



In this case, both A_x and A_y are positive.

(b)

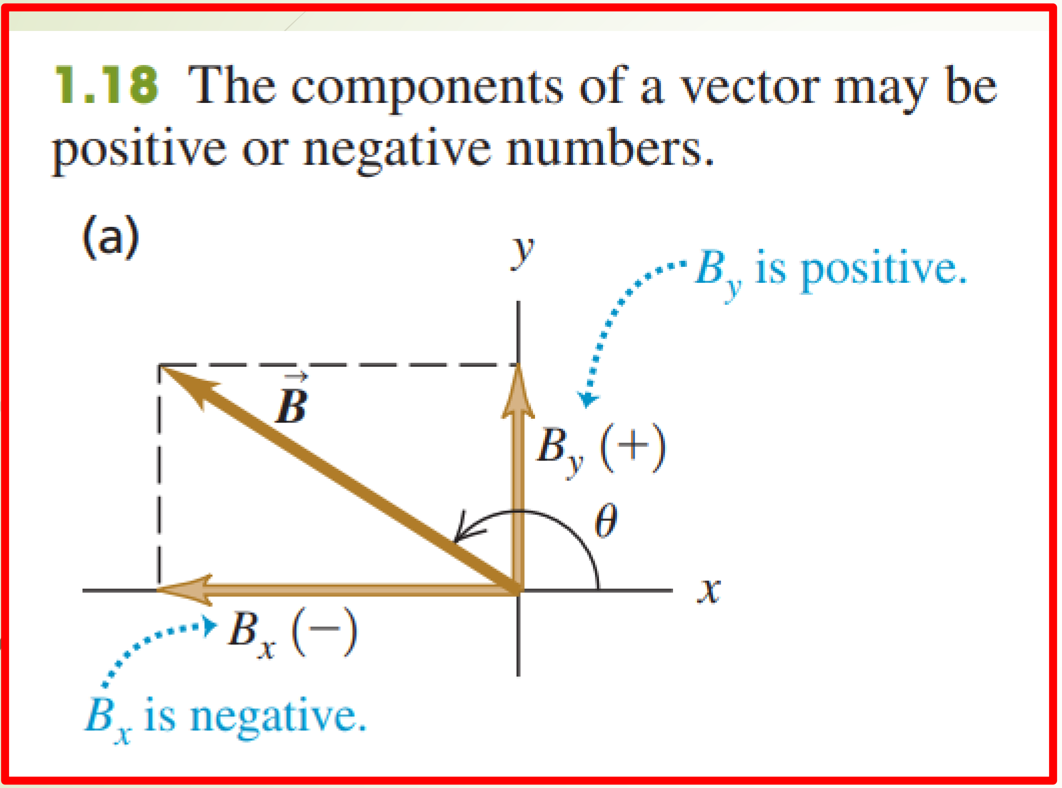


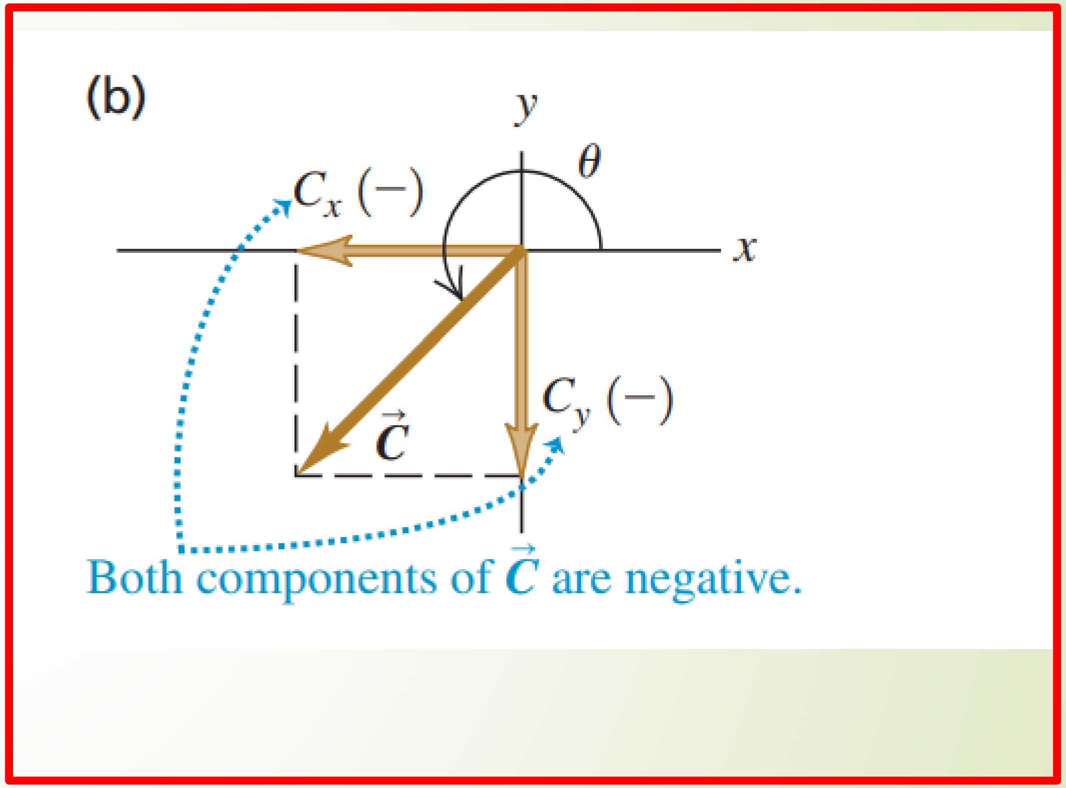
$$\frac{A_x}{A} = \cos\theta \quad \text{and} \quad \frac{A_y}{A} = \sin\theta$$

$$A_x = A\cos\theta \quad \text{and} \quad A_y = A\sin\theta$$

(θ measured from the $+x$-axis, rotating toward the $+y$-axis)

The sign of the components depends on the angle from zero degree;





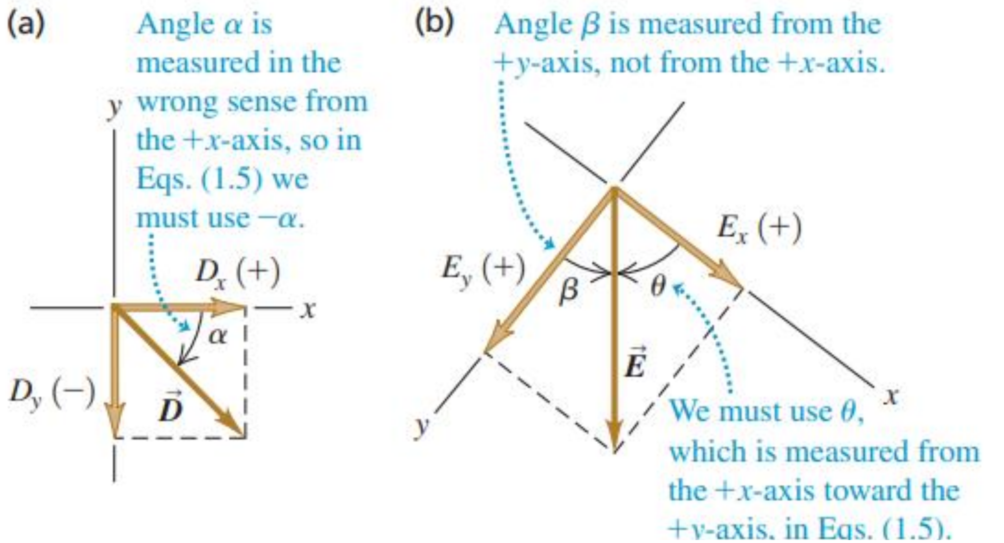
EXAMPLE 1.6 FINDING COMPONENTS

(a) What are the x- and y-components of vector $\vec{D}$ in **Fig. 1.19a**? The magnitude of the vector is $D = 3.00$ m, and angle $\alpha = 45°$. (b) What are the x- and y-components of vector $\vec{E}$ in Fig. 1.19b? The magnitude of the vector is $E = 4.50$ m, and angle $\beta = 37.0°$.

SOLUTION

IDENTIFY and SET UP: We can use Eqs. (1.5) to find the components of these vectors, but we must be careful: Neither angle α nor β in Fig. 1.19 is measured from the $+x$-axis toward the $+y$-axis. We estimate from the figure that the lengths of both

1.19 Calculating the x- and y-components of vectors.



components in part (a) are roughly 2 m, and that those in part (b) are 3 m and 4 m. The figure indicates the signs of the components.

EXECUTE: (a) The angle α (the Greek letter alpha) between the positive x-axis and $\vec{D}$ is measured toward the *negative* y-axis. The angle we must use in Eqs. (1.5) is $\theta = -\alpha = -45°$. We then find

$$D_x = D\cos\theta = (3.00\text{ m})(\cos(-45°)) = +2.1\text{ m}$$

$$D_y = D\sin\theta = (3.00\text{ m})(\sin(-45°)) = -2.1\text{ m}$$

Had we carelessly substituted $+45°$ for θ in Eqs. (1.5), our result for D_y would have had the wrong sign.

(b) The x- and y-axes in Fig. 1.19b are at right angles, so it doesn't matter that they aren't horizontal and vertical, respectively. But we can't use the angle β (the Greek letter beta) in Eqs. (1.5), because β is measured from the $+y$-axis. Instead, we must use the angle $\theta = 90.0° - \beta = 90.0° - 37.0° = 53.0°$. Then we find

$$E_x = E\cos 53.0° = (4.50\text{ m})(\cos 53.0°) = +2.71\text{ m}$$

$$E_y = E\sin 53.0° = (4.50\text{ m})(\sin 53.0°) = +3.59\text{ m}$$

EVALUATE: Our answers to both parts are close to our predictions. But why do the answers in part (a) correctly have only two significant figures?

Using Components to Do Vector Calculations

1) Resolve each vector into its x- and y-components.

$A_x = A\cos\theta \qquad A_y = A\sin\theta$

$B_x = B\cos\theta \qquad B_y = B\sin\theta \qquad$ etc.

2) Add the x-components together to get R_x and the y-components to get R_y.

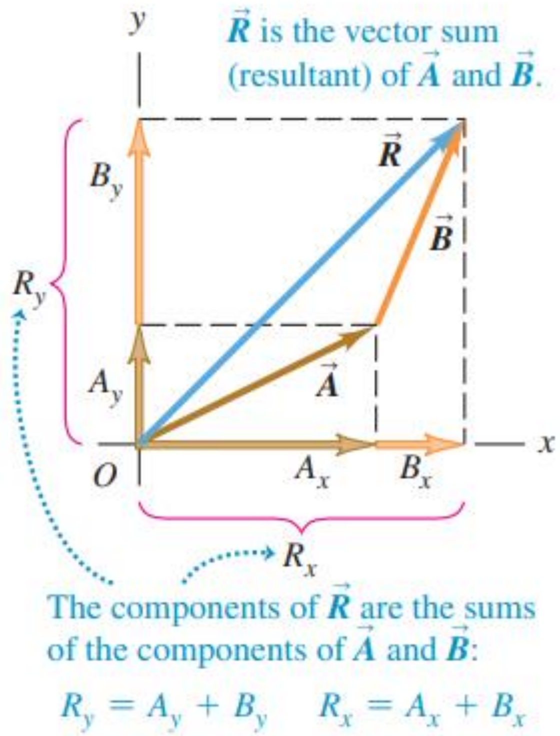
$R_x = A_x + B_x \qquad R_y = A_y + B_y$

3) Calculate the magnitude of the resultant with the Pythagorean Theorem $R = \sqrt{R_x^{\,2} + R_y^{\,2}}$

4) Determine the angle with the equation $\theta = \tan^{-1} |R_y|/|R_x|$.

$$\tan\theta = \frac{A_y}{A_x} \quad \text{and} \quad \theta = \arctan\frac{A_y}{A_x}$$

1.21 Finding the vector sum (resultant) of $\vec{A}$ and $\vec{B}$ using components.



Any two angles that differ by 180° have the same tangent.

We have to look at the individual components.

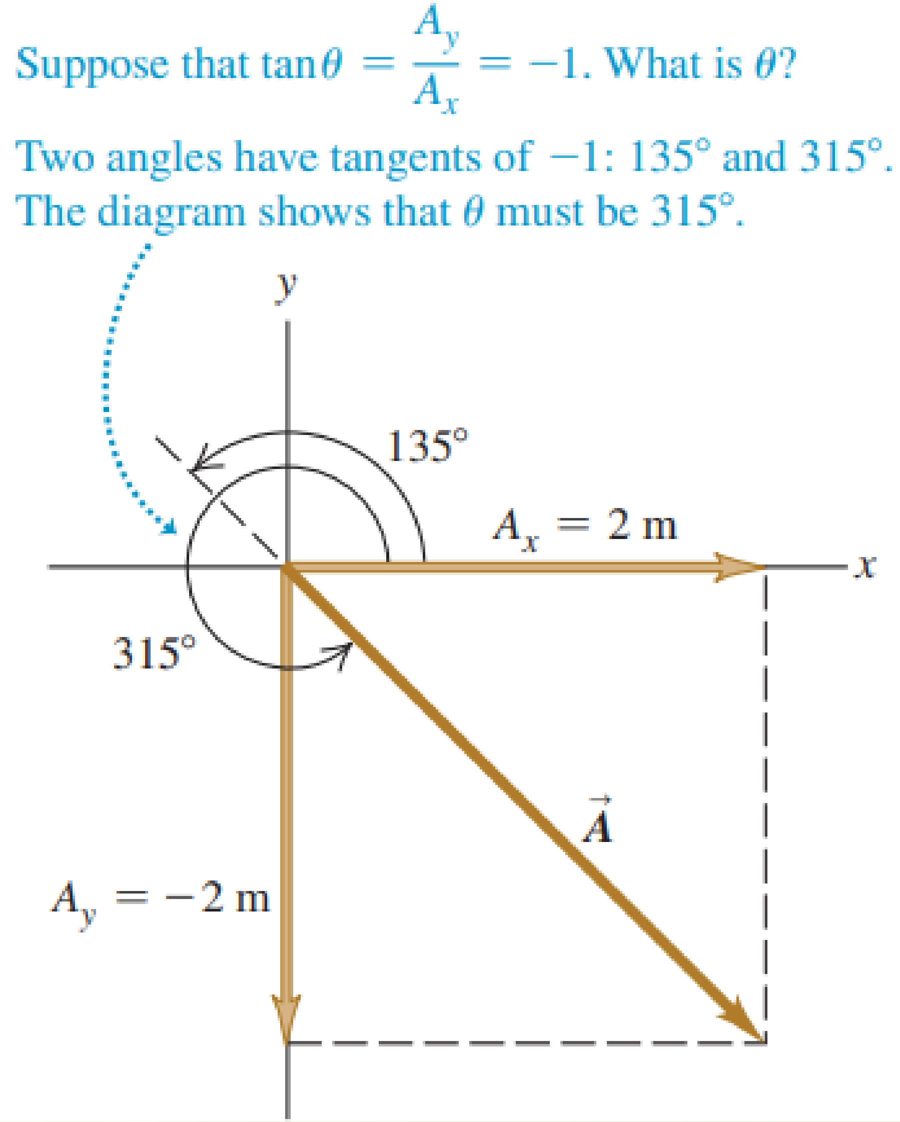
Four cases exist

- Both positive (θ)
- A_x is negative ($180-\theta$)
- Both negative ($\theta+180$)
- A_y is negative ($360-\theta$)

In this example: A_x is positive and A_y is negative, the angle must be in the fourth quadrant

$\theta = 360-45=315^o$

1.20 Drawing a sketch of a vector reveals the signs of its x- and y-components.



EXAMPLE 1.7 USING COMPONENTS TO ADD VECTORS

Three players on a reality TV show are brought to the center of a large, flat field. Each is given a meter stick, a compass, a calculator, a shovel, and (in a different order for each contestant) the following three displacements:

$\vec{A}$: 72.4 m, 32.0° east of north

$\vec{B}$: 57.3 m, 36.0° south of west

$\vec{C}$: 17.8 m due south

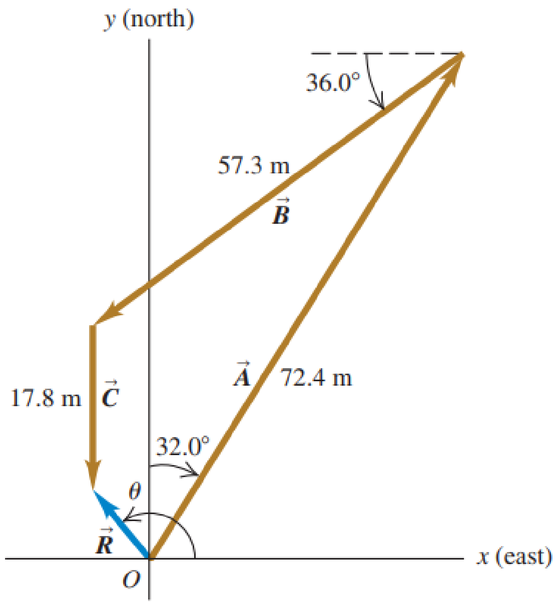
The three displacements lead to the point in the field where the keys to a new Porsche are buried. Two players start measuring immediately, but the winner first *calculates* where to go. What does she calculate?

SOLUTION

IDENTIFY and SET UP: The goal is to find the sum (resultant) of the three displacements, so this is a problem in vector addition. See **Figure 1.23.** We have chosen the $+x$-axis as east and the $+y$-axis as north. We estimate from the diagram that the vector sum $\vec{R}$ is about 10 m, 40° west of north (so θ is about 90° plus 40°, or about 130°).

Continued

1.23 Three successive displacements $\vec{A}$, $\vec{B}$, and $\vec{C}$ and the resultant (vector sum) displacement $\vec{R} = \vec{A} + \vec{B} + \vec{C}$.



EXECUTE: The angles of the vectors, measured from the $+x$-axis toward the $+y$-axis, are $(90.0° - 32.0°) = 58.0°$, $(180.0° + 36.0°) = 216.0°$, and 270.0°, respectively. We may now use Eqs. (1.5) to find the components of $\vec{A}$:

$$A_x = A\cos\theta_A = (72.4\text{ m})(\cos 58.0°) = 38.37\text{ m}$$

$$A_y = A\sin\theta_A = (72.4\text{ m})(\sin 58.0°) = 61.40\text{ m}$$

We've kept an extra significant figure in the components; we'll round to the correct number of significant figures at the end of our calculation. The table below shows the components of all the displacements, the addition of the components, and the other calculations from Eqs. (1.6) and (1.7).

Distance	Angle	x-component	y-component
$A = 72.4$ m	58.0°	38.37 m	61.40 m
$B = 57.3$ m	216.0°	−46.36 m	−33.68 m
$C = 17.8$ m	270.0°	0.00 m	−17.80 m
		$R_x = -7.99$ m	$R_y = 9.92$ m

$$R = \sqrt{(-7.99\text{ m})^2 + (9.92\text{ m})^2} = 12.7\text{ m}$$

$$\theta = \arctan\frac{9.92\text{ m}}{-7.99\text{ m}} = -51°$$

Comparing to angle θ in Fig. 1.23 shows that the calculated angle is clearly off by 180°. The correct value is $\theta = 180° + (-51°) = 129°$, or 39° west of north.

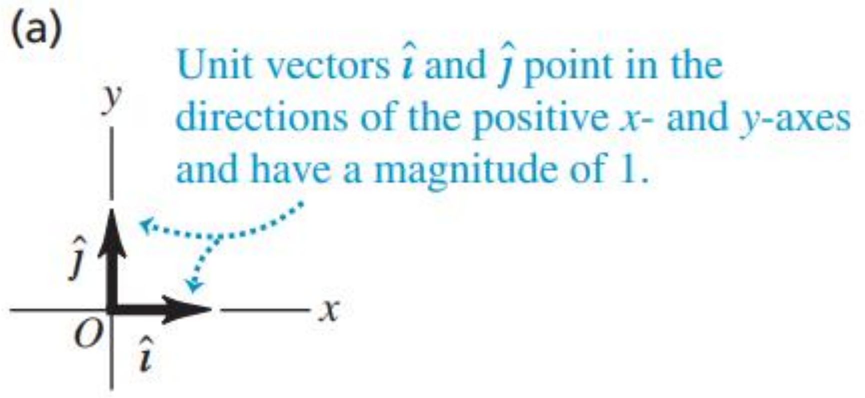
EVALUATE: Our calculated answers for R and θ agree with our estimates. Notice how drawing the diagram in Fig. 1.23 made it easy to avoid a 180° error in the direction of the vector sum.

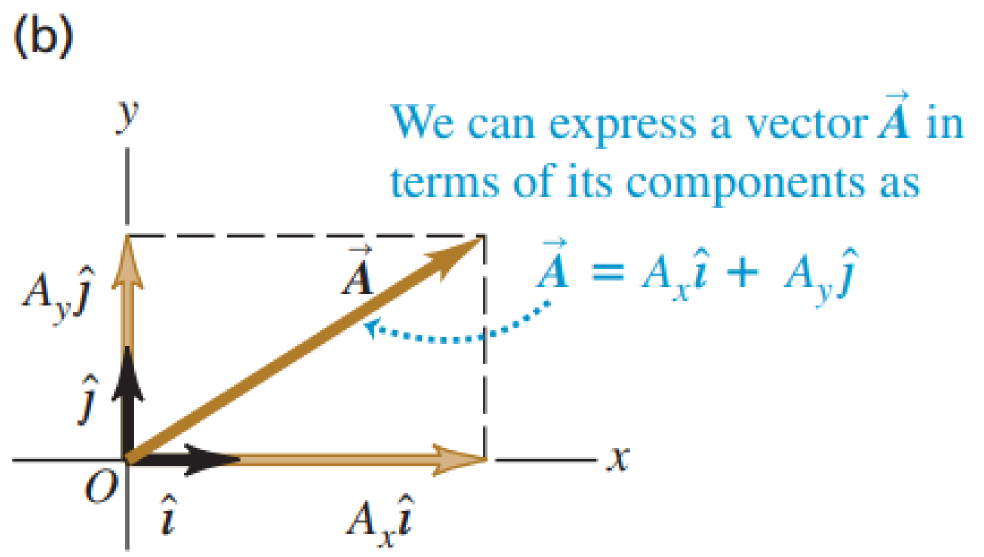
1.9 UNIT VECTORS

A unit vector is a vector that has a magnitude of 1, with no units. Its only purpose is to point that is, to describe a direction in space.

We will always include a caret, or “hat” (^) , in the symbol for a unit vector to distinguish it from ordinary vectors.

1.24 (a) The unit vectors $\hat{\imath}$ and $\hat{\jmath}$.
(b) Expressing a vector $\vec{A}$ in terms of its components.



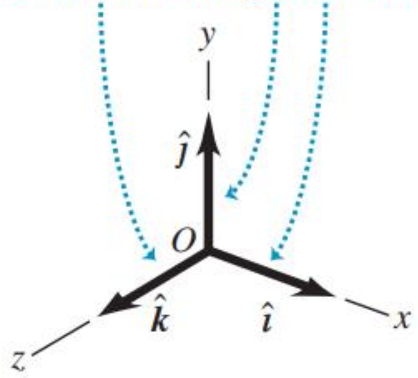


Using unit vectors, we can express the vector sum $\vec{R}$ of two vectors $\vec{A}$ and $\vec{B}$ as follows:

$$\begin{aligned}
\vec{A} &= A_x\hat{\imath} + A_y\hat{\jmath} \\
\vec{B} &= B_x\hat{\imath} + B_y\hat{\jmath} \\
\vec{R} &= \vec{A} + \vec{B} \\
&= (A_x\hat{\imath} + A_y\hat{\jmath}) + (B_x\hat{\imath} + B_y\hat{\jmath}) \\
&= (A_x + B_x)\hat{\imath} + (A_y + B_y)\hat{\jmath} \\
&= R_x\hat{\imath} + R_y\hat{\jmath}
\end{aligned} \tag{1.13}$$

1.25 The unit vectors $\hat{\imath}$, $\hat{\jmath}$, and $\hat{k}$.

Unit vectors $\hat{\imath}$, $\hat{\jmath}$, and $\hat{k}$ point in the directions of the positive x-, y-, and z-axes and have a magnitude of 1.



Any vector can be expressed in terms of its x-, y-, and z-components ...

$$\begin{aligned}
\vec{A} &= A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k} \\
\vec{B} &= B_x\hat{\imath} + B_y\hat{\jmath} + B_z\hat{k}
\end{aligned} \tag{1.14}$$

... and unit vectors $\hat{\imath}$, $\hat{\jmath}$, and $\hat{k}$.

$$\begin{aligned}
\vec{R} &= (A_x + B_x)\hat{\imath} + (A_y + B_y)\hat{\jmath} + (A_z + B_z)\hat{k} \\
&= R_x\hat{\imath} + R_y\hat{\jmath} + R_z\hat{k}
\end{aligned} \tag{1.15}$$

EXAMPLE 1.8 USING UNIT VECTORS

Given the two displacements

$$\vec{D} = (6.00\,\hat{\imath} + 3.00\,\hat{\jmath} - 1.00\hat{k})\text{ m} \quad \text{and}$$

$$\vec{E} = (4.00\,\hat{\imath} - 5.00\,\hat{\jmath} + 8.00\hat{k})\text{ m}$$

find the magnitude of the displacement $2\vec{D} - \vec{E}$.

SOLUTION

IDENTIFY and SET UP: We are to multiply vector $\vec{D}$ by 2 (a scalar) and subtract vector $\vec{E}$ from the result, so as to obtain the vector $\vec{F} = 2\vec{D} - \vec{E}$. Equation (1.8) says that to multiply $\vec{D}$ by 2, we multiply each of its components by 2. We can use Eq. (1.15) to do the subtraction; recall from Section 1.7 that subtracting a vector is the same as adding the negative of that vector.

EXECUTE: We have

$$\begin{aligned}\vec{F} &= 2(6.00\hat{\imath} + 3.00\,\hat{\jmath} - 1.00\hat{k})\text{ m} - (4.00\hat{\imath} - 5.00\,\hat{\jmath} + 8.00\hat{k})\text{ m} \\ &= [(12.00 - 4.00)\hat{\imath} + (6.00 + 5.00)\,\hat{\jmath} + (-2.00 - 8.00)\hat{k}]\text{ m} \\ &= (8.00\hat{\imath} + 11.00\,\hat{\jmath} - 10.00\hat{k})\text{ m}\end{aligned}$$

From Eq. (1.11) the magnitude of $\vec{F}$ is

$$\begin{aligned}F &= \sqrt{F_x^2 + F_y^2 + F_z^2} \\ &= \sqrt{(8.00\text{ m})^2 + (11.00\text{ m})^2 + (-10.00\text{ m})^2} \\ &= 16.9\text{ m}\end{aligned}$$

EVALUATE: Our answer is of the same order of magnitude as the larger components that appear in the sum. We wouldn't expect our answer to be much larger than this, but it could be much smaller.

LUTION

1.10 PRODUCTS OF VECTORS

Scalar (dot) product of vectors $\vec{A}$ and $\vec{B}$

Magnitudes of $\vec{A}$ and $\vec{B}$

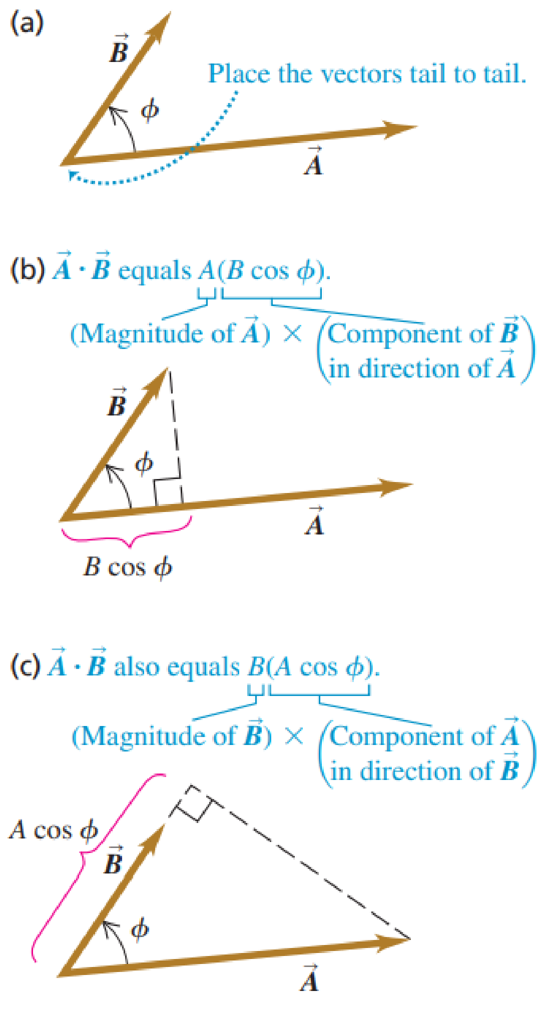
$$\vec{A} \cdot \vec{B} = AB\cos\phi = |\vec{A}||\vec{B}|\cos\phi$$

Angle between $\vec{A}$ and $\vec{B}$ when placed tail to tail

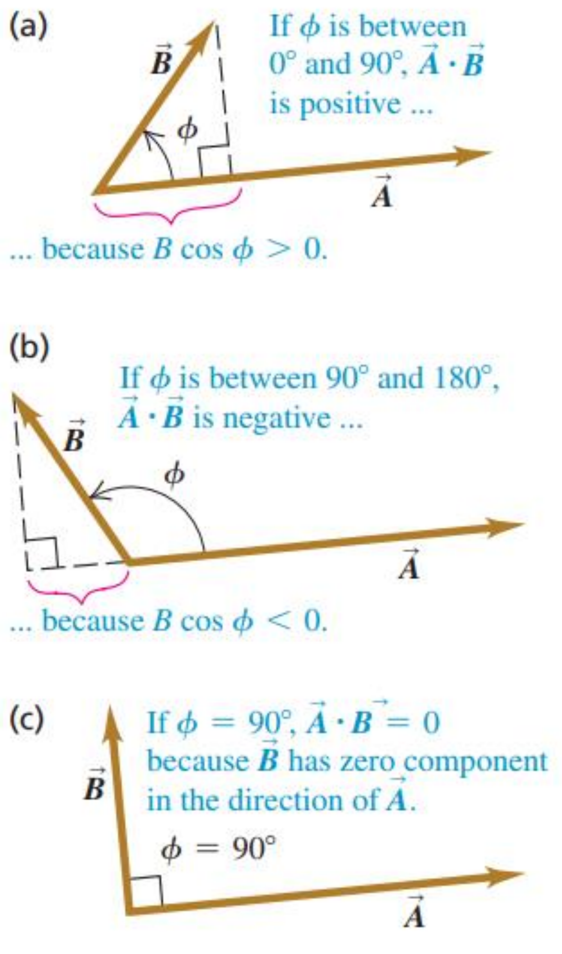
Scalar Product

We denote the **scalar product** of two vectors $\vec{A}$ and $\vec{B}$ by $\vec{A} \cdot \vec{B}$. Because of this notation, the scalar product is also called the **dot product.** Although $\vec{A}$ and $\vec{B}$ are vectors, the quantity $\vec{A} \cdot \vec{B}$ is a scalar.

1.26 Calculating the scalar product of two vectors, $\vec{A} \cdot \vec{B} = AB\cos\phi$.



1.27 The scalar product $\vec{A} \cdot \vec{B} = AB\cos\phi$ can be positive, negative, or zero, depending on the angle between $\vec{A}$ and $\vec{B}$.

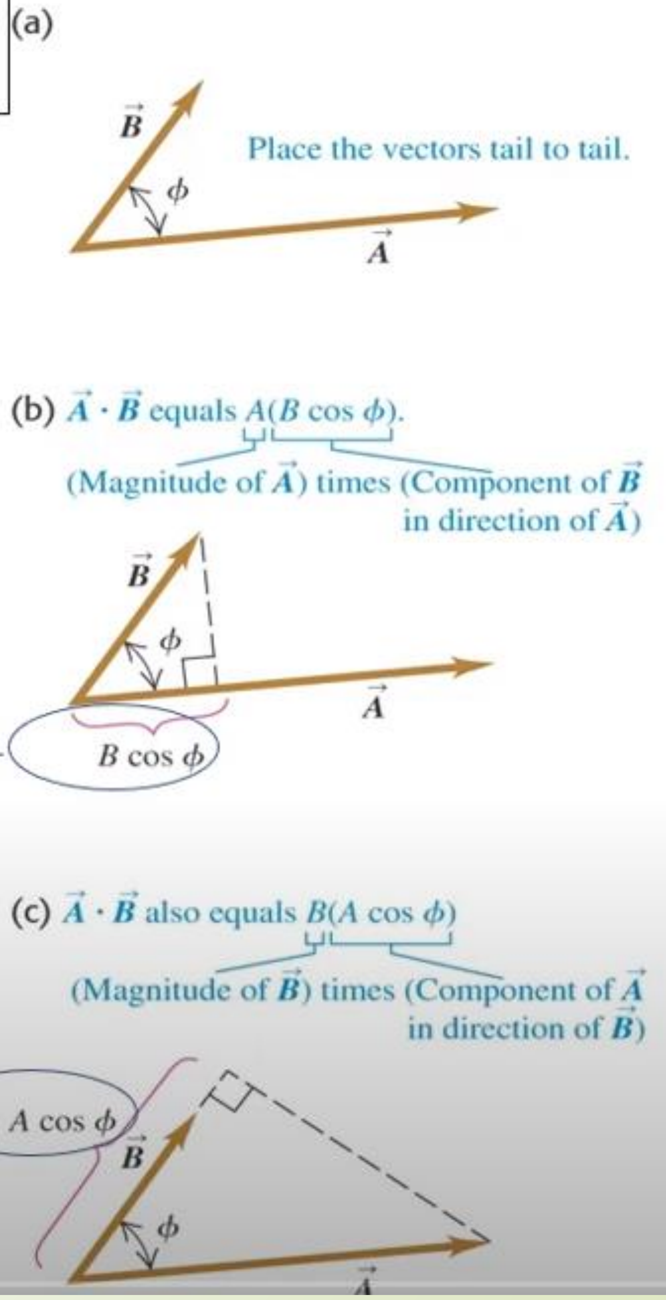


Calculating the scalar product of two vectors:

$$\boxed{C = \vec{A} \bullet \vec{B} = |A| \cdot |B| \cos\phi}$$

$\vec{A} \cdot \vec{B} = A(B\cos\phi)$ = A x ($B_{//}$: the component of B parallel to A)

$\vec{A} \cdot \vec{B} = (A\cos\phi)\,B$ = ($A_{//}$: the component of A parallel to B) x B



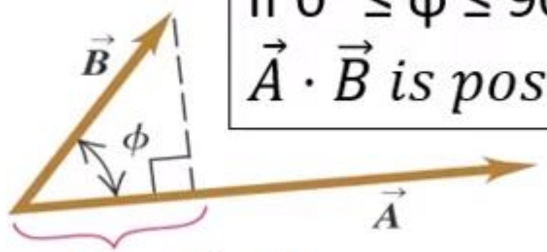
The sign of the scalar product

- When a constant force **F** is applied to a body that undergoes a displacement **d**, the work done by the force is a scalar product, given by $W = \vec{F} \cdot \vec{d}$

The work done by the force is

- <u>Positive</u> if the angle between *F* and ***d*** is between 0 and 90° (example: lifting weight)
- <u>Negative</u> if the angle between *F* and ***d*** is between 90° and 180° (example: stop a moving car)
- <u>Zero</u> and *F* and ***d*** are perpendicular to each other (example: waiter holding a tray of food while walk around)

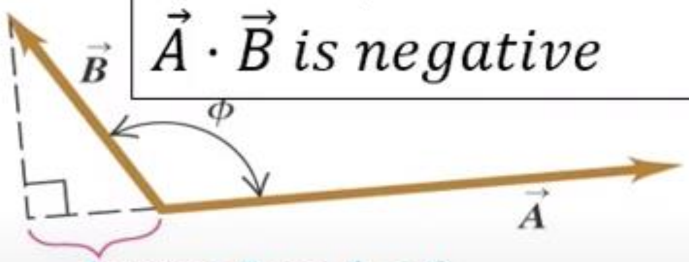
(a)



If $0° \leq \phi \leq 90°$
$\vec{A} \cdot \vec{B}$ *is positive*

... because $B \cos \phi > 0$.

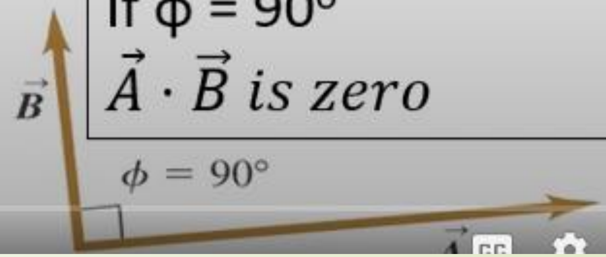
(b)



If $90° \leq \phi \leq 180°$
$\vec{A} \cdot \vec{B}$ *is negative*

... because $B \cos \phi < 0$.

(c)



If $\phi = 90°$
$\vec{A} \cdot \vec{B}$ *is zero*

Using Components to Calculate the Scalar Product

$$\hat{\imath}\cdot\hat{\imath} = \hat{\jmath}\cdot\hat{\jmath} = \hat{k}\cdot\hat{k} = (1)(1)\cos 0^\circ = 1$$

$$\hat{\imath}\cdot\hat{\jmath} = \hat{\imath}\cdot\hat{k} = \hat{\jmath}\cdot\hat{k} = (1)(1)\cos 90^\circ = 0$$

$$\begin{aligned}
\vec{A}\cdot\vec{B} &= (A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k})\cdot(B_x\hat{\imath} + B_y\hat{\jmath} + B_z\hat{k}) \\
&= A_x\hat{\imath}\cdot B_x\hat{\imath} + A_x\hat{\imath}\cdot B_y\hat{\jmath} + A_x\hat{\imath}\cdot B_z\hat{k} \\
&\quad + A_y\hat{\jmath}\cdot B_x\hat{\imath} + A_y\hat{\jmath}\cdot B_y\hat{\jmath} + A_y\hat{\jmath}\cdot B_z\hat{k} \\
&\quad + A_z\hat{k}\cdot B_x\hat{\imath} + A_z\hat{k}\cdot B_y\hat{\jmath} + A_z\hat{k}\cdot B_z\hat{k} \\
&= A_xB_x\hat{\imath}\cdot\hat{\imath} + A_xB_y\hat{\imath}\cdot\hat{\jmath} + A_xB_z\hat{\imath}\cdot\hat{k} \\
&\quad + A_yB_x\hat{\jmath}\cdot\hat{\imath} + A_yB_y\hat{\jmath}\cdot\hat{\jmath} + A_yB_z\hat{\jmath}\cdot\hat{k} \\
&\quad + A_zB_x\hat{k}\cdot\hat{\imath} + A_zB_y\hat{k}\cdot\hat{\jmath} + A_zB_z\hat{k}\cdot\hat{k}
\end{aligned}$$

Scalar (dot) product of vectors $\vec{A}$ and $\vec{B}$

Components of $\vec{A}$

$$\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y + A_zB_z$$

Components of $\vec{B}$

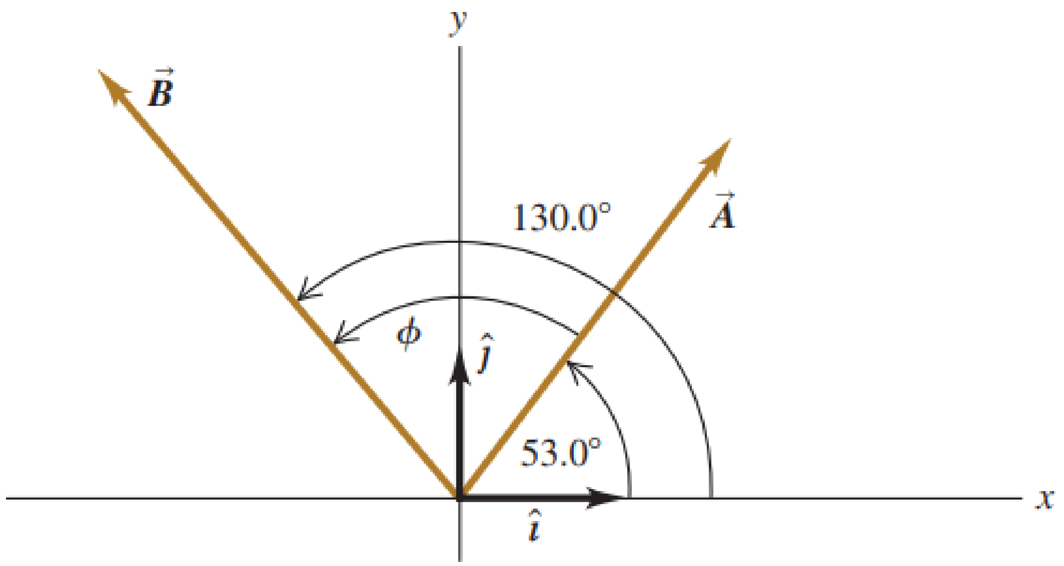
EXAMPLE 1.9 CALCULATING A SCALAR PRODUCT

Find the scalar product $\vec{A} \cdot \vec{B}$ of the two vectors in **Fig. 1.28.** The magnitudes of the vectors are $A = 4.00$ and $B = 5.00$.

SOLUTION

IDENTIFY and SET UP: We can calculate the scalar product in two ways: using the magnitudes of the vectors and the angle between them (Eq. 1.16), and using the components of the vectors (Eq. 1.19). We'll do it both ways, and the results will check each other.

1.28 Two vectors $\vec{A}$ and $\vec{B}$ in two dimensions.



EXECUTE: The angle between the two vectors $\vec{A}$ and $\vec{B}$ is $\phi = 130.0° - 53.0° = 77.0°$, so Eq. (1.16) gives us

$$\vec{A} \cdot \vec{B} = AB \cos \phi = (4.00)(5.00) \cos 77.0° = 4.50$$

To use Eq. (1.19), we must first find the components of the vectors. The angles of $\vec{A}$ and $\vec{B}$ are given with respect to the $+x$-axis and are measured in the sense from the $+x$-axis to the $+y$-axis, so we can use Eqs. (1.5):

$$A_x = (4.00) \cos 53.0° = 2.407$$
$$A_y = (4.00) \sin 53.0° = 3.195$$
$$B_x = (5.00) \cos 130.0° = -3.214$$
$$B_y = (5.00) \sin 130.0° = 3.830$$

As in Example 1.7, we keep an extra significant figure in the components and round at the end. Equation (1.19) now gives us

$$\vec{A} \cdot \vec{B} = A_xB_x + A_yB_y + A_zB_z$$
$$= (2.407)(-3.214) + (3.195)(3.830) + (0)(0) = 4.50$$

EVALUATE: Both methods give the same result, as they should.

EXAMPLE 1.10 FINDING AN ANGLE WITH THE SCALAR PRODUCT

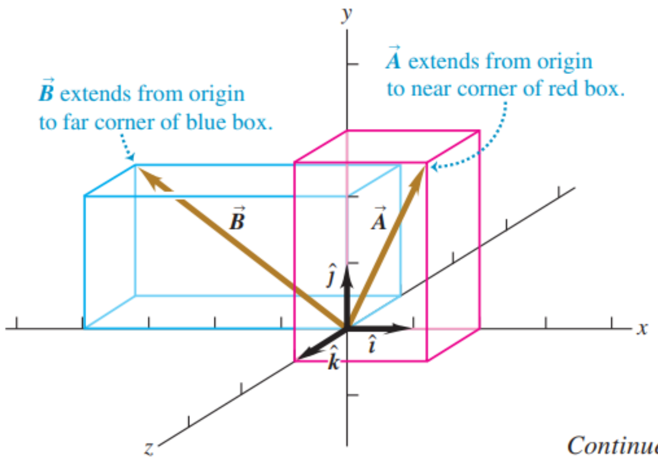
Find the angle between the vectors

$$\vec{A} = 2.00\hat{\imath} + 3.00\hat{\jmath} + 1.00\hat{k}$$

and

$$\vec{B} = -4.00\hat{\imath} + 2.00\hat{\jmath} - 1.00\hat{k}$$

1.29 Two vectors in three dimensions.



Continued

SOLUTION

IDENTIFY and SET UP: We're given the x-, y-, and z-components of two vectors. Our target variable is the angle ϕ between them (**Fig. 1.29**). To find this, we'll solve Eq. (1.16), $\vec{A}\cdot\vec{B} = AB\cos\phi$, for ϕ in terms of the scalar product $\vec{A}\cdot\vec{B}$ and the magnitudes A and B. We can use Eq. (1.19) to evaluate the scalar product, $\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y + A_zB_z$, and we can use Eq. (1.6) to find A and B.

EXECUTE: We solve Eq. (1.16) for $\cos\phi$ and use Eq. (1.19) to write $\vec{A}\cdot\vec{B}$:

$$\cos\phi = \frac{\vec{A}\cdot\vec{B}}{AB} = \frac{A_xB_x + A_yB_y + A_zB_z}{AB}$$

We can use this formula to find the angle between *any* two vectors $\vec{A}$ and $\vec{B}$. Here we have $A_x = 2.00$, $A_y = 3.00$, and $A_z = 1.00$, and $B_x = -4.00$, $B_y = 2.00$, and $B_z = -1.00$. Thus

$$\begin{aligned}\vec{A}\cdot\vec{B} &= A_xB_x + A_yB_y + A_zB_z\\ &= (2.00)(-4.00) + (3.00)(2.00) + (1.00)(-1.00)\\ &= -3.00\end{aligned}$$

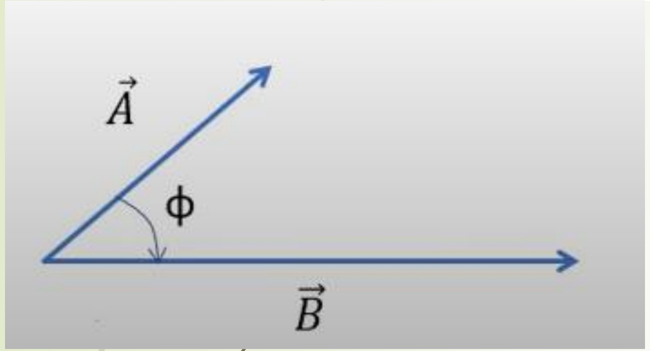
$$\begin{aligned}A &= \sqrt{A_x^2 + A_y^2 + A_z^2} = \sqrt{(2.00)^2 + (3.00)^2 + (1.00)^2}\\ &= \sqrt{14.00}\\ B &= \sqrt{B_x^2 + B_y^2 + B_z^2} = \sqrt{(-4.00)^2 + (2.00)^2 + (-1.00)^2}\\ &= \sqrt{21.00}\\ \cos\phi &= \frac{A_xB_x + A_yB_y + A_zB_z}{AB} = \frac{-3.00}{\sqrt{14.00}\,\sqrt{21.00}} = -0.175\\ \phi &= 100^\circ\end{aligned}$$

EVALUATE: As a check on this result, note that the scalar product $\vec{A}\cdot\vec{B}$ is negative. This means that ϕ is between 90° and 180° (see Fig. 1.27), which agrees with our answer.

Vector Product

The vector product of the two vectors, also called the cross product.

$\vec{A}$ and $\vec{B}$ → $\vec{A} \times \vec{B}$



The vector product of; $\vec{C}$

$$\vec{C} = \vec{A} \times \vec{B}$$

The magnitude; $\vec{C}$

$$|C| = AB \sin\varphi$$

The direction perpendicular to the plane stablished by the two vectors $\vec{A}$ and $\vec{B}$

Magnitude of **vector (cross) product** of vectors $\vec{B}$ and $\vec{A}$

$$C = AB \sin\phi \qquad (1.20)$$

Magnitudes of $\vec{A}$ and $\vec{B}$

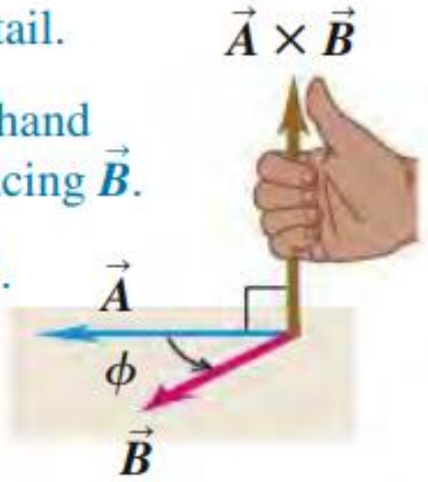
Angle between $\vec{A}$ and $\vec{B}$ when placed tail to tail

Right hand rule

To find the direction of the $\vec{A} \times \vec{B}$

(a) Using the right-hand rule to find the direction of $\vec{A} \times \vec{B}$

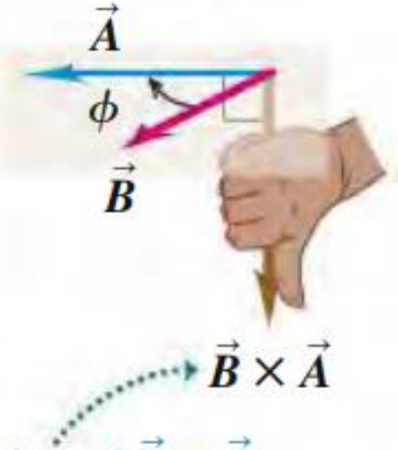
1. Place $\vec{A}$ and $\vec{B}$ tail to tail.
2. Point fingers of right hand along $\vec{A}$, with palm facing $\vec{B}$.
3. Curl fingers toward $\vec{B}$.
4. Thumb points in direction of $\vec{A} \times \vec{B}$.



To find the direction of the $\vec{B} \times \vec{A}$

(b) Using the right-hand rule to find the direction of $\vec{B} \times \vec{A} = -\vec{A} \times \vec{B}$ (vector product is anticommutative)

1. Place $\vec{B}$ and $\vec{A}$ tail to tail.
2. Point fingers of right hand along $\vec{B}$, with palm facing $\vec{A}$.
3. Curl fingers toward $\vec{A}$.
4. Thumb points in direction of $\vec{B} \times \vec{A}$.
5. $\vec{B} \times \vec{A}$ has same magnitude as $\vec{A} \times \vec{B}$ but points in opposite direction.



Magnitude the vector product of two vectors

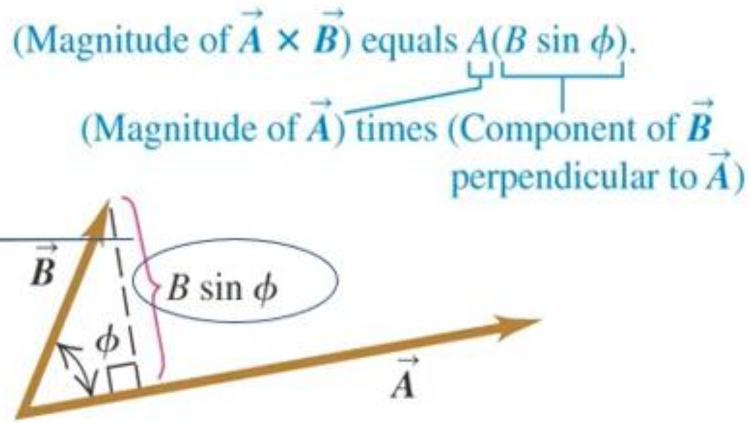
$C = AB \sin \phi$

ϕ is the **smaller** of the two possible angles.
Since $0 \le \phi \le 180^o$, $0 \le \sin\phi \le 1$, $|\boldsymbol{A} \times \boldsymbol{B}|$ is **never negative.**

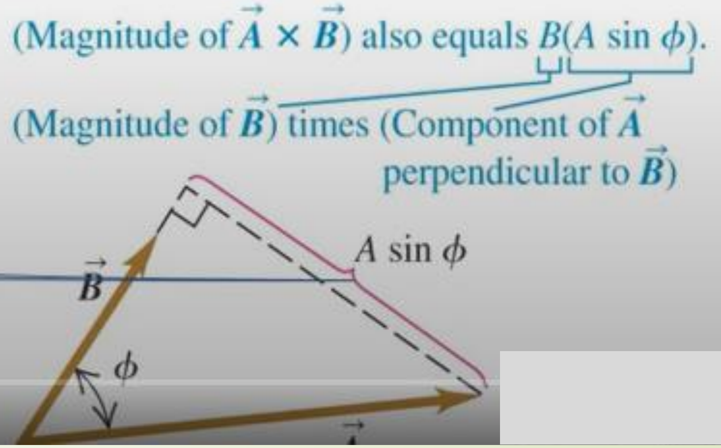
C = A(B sin ϕ) = A x ($B_\perp$: the component of B perpendicular to A)

C = Asin ϕ (B) = ($A_\perp$: the component of A perpendicular to B) x B

(a)



(b)

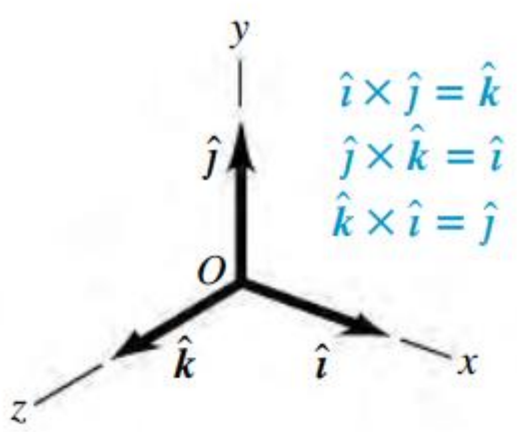


Using Components to Calculate the Vector Product

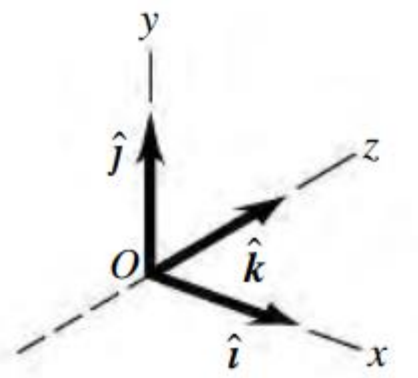
$$\hat{\imath} \times \hat{\imath} = \hat{\jmath} \times \hat{\jmath} = \hat{k} \times \hat{k} = 0$$

$$\hat{\imath} \times \hat{\jmath} = -\hat{\jmath} \times \hat{\imath} = \hat{k}$$
$$\hat{\jmath} \times \hat{k} = -\hat{k} \times \hat{\jmath} = \hat{\imath}$$
$$\hat{k} \times \hat{\imath} = -\hat{\imath} \times \hat{k} = \hat{\jmath}$$

(a) A right-handed coordinate system



(b) A left-handed coordinate system; we will not use these.



$$\vec{A} \times \vec{B} = (A_x \hat{\imath} + A_y \hat{\jmath} + A_z \hat{k}) \times (B_x \hat{\imath} + B_y \hat{\jmath} + B_z \hat{k})$$

$$= A_x \hat{\imath} \times B_x \hat{\imath} + A_x \hat{\imath} \times B_y \hat{\jmath} + A_x \hat{\imath} \times B_z \hat{k}$$

$$+ A_y \hat{\jmath} \times B_x \hat{\imath} + A_y \hat{\jmath} \times B_y \hat{\jmath} + A_y \hat{\jmath} \times B_z \hat{k}$$

$$+ A_z \hat{k} \times B_x \hat{\imath} + A_z \hat{k} \times B_y \hat{\jmath} + A_z \hat{k} \times B_z \hat{k}$$

$$\vec{A}\mathbf{x}\vec{B} = (A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k})x(B_x\hat{\imath} + B_y\hat{\jmath} + B_z\hat{k})$$

$$= \cancel{A_x\hat{\imath}xB_x\hat{\imath}} + A_x\hat{\imath}xB_y\hat{\jmath} + A_x\hat{\imath}xB_z\hat{k}$$

$$+ A_y\hat{\jmath}xB_x\hat{\imath} + \cancel{A_y\hat{\jmath}xB_y\hat{\jmath}} + A_y\hat{\jmath}xB_z\hat{k}$$

$$+ A_z\hat{k}xB_x\hat{\imath} + A_z\hat{k}xB_y\hat{\jmath} + \cancel{A_z\hat{k}xB_z\hat{k}}$$

$$\boxed{\vec{A} \text{ x } \vec{B} = (A_yB_z - A_zB_y)\,\hat{\imath} + (A_zB_x - A_xB_z)\,\hat{\jmath} + (A_xB_y - A_yB_x)\,\hat{k}}$$

If $\vec{\boldsymbol{C}} = \vec{\boldsymbol{A}} \text{ x } \vec{\boldsymbol{B}}$ then

$C_x = A_yB_z - A_zB_y$; $\quad C_y = A_zB_x - A_xB_z$; $\quad C_z = A_xB_y - A_yB_x$

$$\vec{A} \times \vec{B} = (A_yB_z - A_zB_y)\hat{\imath} + (A_zB_x - A_xB_z)\hat{\jmath} + (A_xB_y - A_yB_x)\hat{k} \qquad (1.24)$$

If you compare Eq. (1.24) with Eq. (1.14), you'll see that the components of $\vec{C} = \vec{A} \times \vec{B}$ are

Components of vector (cross) product $\vec{A} \times \vec{B}$

$$C_x = A_yB_z - A_zB_y \qquad C_y = A_zB_x - A_xB_z \qquad C_z = A_xB_y - A_yB_x \qquad (1.25)$$

A_x, A_y, A_z = components of $\vec{A}$ $\qquad$ B_x, B_y, B_z = components of $\vec{B}$

The vector product can also be expressed in determinant form as

$\vec{A} \times \vec{B} =$

i	***j***	***k***	***i***	***j***	***k***
A_x	A_y	A_z	A_x	A_y	A_z
B_x	B_y	B_z	B_x	B_y	B_z

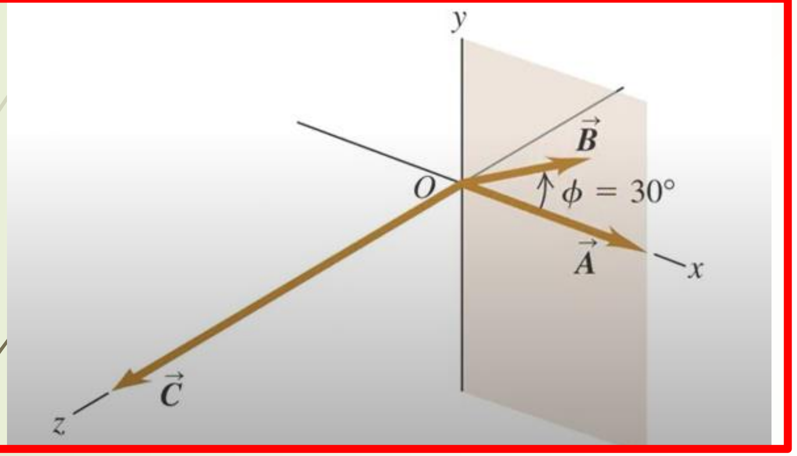
- direction

+ direction

$$\vec{A} \times \vec{B} = (A_yB_z - A_zB_y)\,\boldsymbol{i} + (A_zB_x - A_xB_z)\,\boldsymbol{j} + (A_xB_y - A_yB_x)\,\boldsymbol{k}$$

EXAMPLE 1.11 CALCULATING A VECTOR PRODUCT

Vector $\vec{A}$ has magnitude 6 units and is in the direction of the $+x$-axis. Vector $\vec{B}$ has magnitude 4 units and lies in the xy-plane, making an angle of 30° with the $+x$-axis (**Fig. 1.33**). Find the vector product $\vec{C} = \vec{A} \times \vec{B}$.



SOLUTION

IDENTIFY and SET UP: We'll find the vector product in two ways, which will provide a check of our calculations. First we'll use Eq. (1.20) and the right-hand rule; then we'll use Eqs. (1.25) to find the vector product by using components.

1.33 Vectors $\vec{A}$ and $\vec{B}$ and their vector product $\vec{C} = \vec{A} \times \vec{B}$. Vector $\vec{B}$ lies in the xy-plane.

EXECUTE: From Eq. (1.20) the magnitude of the vector product is

$$AB \sin\phi = (6)(4)(\sin 30°) = 12$$

By the right-hand rule, the direction of $\vec{A} \times \vec{B}$ is along the $+z$-axis (the direction of the unit vector $\hat{k}$), so $\vec{C} = \vec{A} \times \vec{B} = 12\hat{k}$.

To use Eqs. (1.25), we first determine the components of $\vec{A}$ and $\vec{B}$. Note that $\vec{A}$ points along the x-axis, so its only nonzero component is A_x. For $\vec{B}$, Fig. 1.33 shows that $\phi = 30°$ is measured from the $+x$-axis toward the $+y$-axis, so we can use Eqs. (1.5):

$$A_x = 6 \qquad A_y = 0 \qquad A_z = 0$$

$$B_x = 4\cos 30° = 2\sqrt{3} \qquad B_y = 4\sin 30° = 2 \qquad B_z = 0$$

Then Eqs. (1.25) yield

$$C_x = (0)(0) - (0)(2) = 0$$

$$C_y = (0)(2\sqrt{3}) - (6)(0) = 0$$

$$C_z = (6)(2) - (0)(2\sqrt{3}) = 12$$

Thus again we have $\vec{C} = 12\hat{k}$.

EVALUATE: Both methods give the same result. Depending on the situation, one or the other of the two approaches may be the more convenient one to use.


Physics
THANK YOU