

R1, R2:- Fractions & Decimal Numbers

Fraction: A fraction is a number of the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$

In any fraction $\frac{p}{q}$, the number p is the **numerator** and the number q is the **denominator**.

$$\text{fraction} = \frac{\text{numerator}}{\text{denominator}} = \frac{p}{q}$$

$$\text{Ex: } \frac{3}{5}, -\frac{5}{8}, \frac{22}{7}, \frac{5}{1} = 5 \text{ etc.}$$

Decimal Fraction:- A fraction whose denominator is a power of 10 and its numerator is the figures (digits) placed to the right of a decimal point.

$$\text{Ex: - } 0.123 = \frac{123}{1000}$$

$$0.4 = \frac{4}{10}$$

$$\frac{53}{100} = 0.53$$

Proper Fraction: In a fraction $\frac{p}{q}$, the number p is less than q , then it is a proper fraction. Ex: $\frac{1}{2}, \frac{4}{7}, \frac{3}{4}$ etc.

- The value of a proper fraction is less than 1.
- Also the fractions $-\frac{1}{2}, -\frac{4}{7}, -\frac{3}{4}$ etc. are proper fractions (ignore the negative sign).

Improper Fraction: In a fraction $\frac{p}{q}$, if the number p is greater than or equal to q , then it is an improper fraction. Ex: $\frac{11}{5}, \frac{7}{7}, \frac{21}{4}$ etc.

❖ The number of decimal digits indicates the number of zeros in the denominator.

Ex: - $.6 = \frac{6}{10}$ -----> One decimal digit; one 0 in the denominator.

$.06 = \frac{6}{100}$ ----> Two decimal digits; two 0's in the denominator.

$.006 = \frac{6}{1000}$ ---> Three decimal digits; three 0's in the denominator.

Changing Fractions to Decimals

1. Write the following fractions as decimals

a) $\frac{7}{10} = 0.7$ b) $\frac{43}{100} = 0.43$

c) $\frac{51}{1000} = 0.051$

2. Write as a decimal: $\frac{615}{100,000}$

Ans: - Five 0's in the denominator indicate five digits after the decimal

point. Therefore, $\frac{615}{100,000} = .00615$

3. Write this mixed number as a decimal: $5\frac{23}{100}$

Ans: - The whole number 5 does not change. We simply replace the common fraction $\frac{23}{100}$ with the decimal .23.

$$\text{Therefore, } 5\frac{23}{100} = 5.23$$

4. Write this mixed number with a common fraction: 9.0012

Ans: - The whole number 9 does not change. We simply replace the decimal .0012 with the common fraction $\frac{12}{10,000}$.

$$\therefore 9.0012 = 9\frac{12}{10,000}$$

❖ If the denominator is not a power of 10, we make it a power of 10 by multiplying it or dividing it.

5. Write $\frac{8}{25}$ as a decimal.

Ans: Multiplying by 4, we can make the denominator 25 as a power of 10

$$\therefore \frac{8}{25} = \frac{8 \times 4}{25 \times 4} = \frac{32}{100} = 0.32$$

6. Write $\frac{4}{5}$ as a decimal

Ans: $\frac{4}{5} = \frac{8}{10} = .8$ (Multiplying by 2)

7. Write as a decimal $\frac{7}{200}$

Ans: $\frac{7}{200} = \frac{7}{200} \times \frac{5}{5} = \frac{35}{1000} = .035$

8. Write as a decimal $\frac{8}{200}$

Ans: $\frac{8}{200} = \frac{8 \div 2}{200 \div 2} = \frac{4}{100} = .04$ (Dividing by 2)

Changing Decimals to Fractions

1. Change 0.459 into fraction

Ans: $0.459 = \frac{459}{1000}$

2. Change 0.35 into fraction

Ans: $0.35 = \frac{35}{100} = \frac{35 \div 5}{100 \div 5} = \frac{7}{20}$

3. Change 3.25 into fraction

Ans: $3.25 = 3 \frac{25}{100} = 3 \frac{25 \div 25}{100 \div 25} = 3 \frac{1}{4}$

One Number as a fraction of another

1. Write 55 as a fraction of 80.

Ans: $\frac{55}{80} = \frac{55 \div 5}{80 \div 5} = \frac{11}{16}$

2. Write 4 mm as a fraction of 8 cm.

Ans: 8 cm = 80 mm

$$4\text{mm as a fraction of } 80\text{ mm} = \frac{4}{80} = \frac{4 \div 4}{80 \div 4} = \frac{1}{20} \quad \Rightarrow \therefore 4\text{ mm} = \frac{1}{20} \text{ of } 8\text{ cm.}$$

Simplest Form: A fraction is in its simplest form when there are no common factors to both numerator and denominator.

For example, $\frac{16}{20} = \frac{2 \times 2 \times 2 \times 2}{2 \times 2 \times 5} = \frac{4}{5}$

So $\frac{4}{5}$ is the simplest form of the fraction $\frac{16}{20}$

Examples:

R.1.1: - Express $\frac{24}{36}$ in its simplest form

$$\frac{24}{36} = \frac{24 \div 12}{36 \div 12} = \frac{2}{3} \quad \text{OR} \quad \frac{24}{36} = \frac{2 \times 2 \times 2 \times 3}{2 \times 2 \times 3 \times 3} = \frac{2}{3}$$

R.1.3: - Express $\frac{3}{4}$ as an equivalent fraction having a denominator 20.

$$\frac{3}{4} = \frac{3 \times 5}{4 \times 5} = \frac{15}{20}$$

R.1.4: - Express 7 as an equivalent fraction with a denominator 3.

$$7 = \frac{7}{1} = \frac{7 \times 3}{1 \times 3} = \frac{21}{3}$$

Addition and Subtraction of Fractions

Ex: -

1. Add a) $\frac{2}{7} + \frac{4}{7} = \frac{6}{7}$

b) $\frac{5}{8} + \frac{3}{7} = \frac{5 \times 7}{8 \times 7} + \frac{3 \times 8}{7 \times 8} = \frac{35}{56} + \frac{24}{56} = \frac{59}{56}$

2. Subtract a) $\frac{7}{8} - \frac{1}{5} = \frac{7 \times 5}{8 \times 5} - \frac{1 \times 8}{5 \times 8} = \frac{35}{40} - \frac{8}{40} = \frac{27}{40}$

b) $\frac{13}{25} - \frac{4}{25} = \frac{9}{25}$

R.1.6: - Find $\frac{4}{9} - \frac{1}{2} + \frac{5}{6}$

$$= \frac{4 \times 2}{9 \times 2} - \frac{1 \times 9}{2 \times 9} + \frac{5 \times 3}{6 \times 3} = \frac{8}{18} - \frac{9}{18} + \frac{15}{18} = \frac{14 \div 2}{18 \div 2} = \frac{7}{9}$$

R.1.8: - (a) Express $4\frac{2}{5}$ as an improper fraction

$$4\frac{2}{5} = 4 + \frac{2}{5} = \frac{20}{5} + \frac{2}{5} = \frac{22}{5}$$

(b) Find $4\frac{2}{5} + \frac{1}{3}$

$$= \frac{22}{5} + \frac{1}{3} = \frac{22 \times 3}{5 \times 3} + \frac{1 \times 5}{3 \times 5} = \frac{66}{15} + \frac{5}{15} = \frac{71}{15} \text{ (OR) } 4\frac{11}{15}$$

Mixed Fraction: A fraction consisting of a whole number part and a fractional part is a mixed fraction.

- ❖ $2\frac{3}{5}$, $1\frac{1}{7}$ etc. are mixed fractions.
- ❖ In the fraction $2\frac{3}{5}$, the number 2 is whole number part and $\frac{3}{5}$ is fractional part.
- ❖ Mixed fraction can be converted as improper fraction.

Ex. $2\frac{3}{5} = 2 + \frac{3}{5} = \frac{10}{5} + \frac{3}{5} = \frac{13}{5}$

Multiplication of Fractions

To multiply fractions, multiply the numerators and multiply the denominators.

Ex:- $\frac{4}{7} \times \frac{2}{11} = \frac{4 \times 2}{7 \times 11} = \frac{8}{77}$

R.1.9: - Find $\frac{4}{9} \times \frac{3}{8} = \frac{4 \div 4}{9 \div 3} \times \frac{3 \div 3}{8 \div 4} = \frac{1}{3} \times \frac{1}{2} = \frac{1}{6}$ OR $\frac{4}{9} \times \frac{3}{8} = \frac{12 \div 12}{72 \div 12} = \frac{1}{6}$

R.1.10: - Find $\frac{12}{25} \times \frac{2}{7} \times \frac{10}{9} = \frac{\cancel{12}^4}{\cancel{25}_5} \times \frac{2}{7} \times \frac{\cancel{10}^2}{\cancel{9}_3} = \frac{16}{105}$

R.1.11: - Find $\frac{3}{4}$ of $\frac{5}{9} = \frac{\cancel{3}^1}{4} \times \frac{5}{\cancel{9}_3} = \frac{5}{12}$

R.1.12: - Find $\frac{3}{8}$ of 70 $= \frac{3}{8 \div 2} \times \frac{70 \div 2}{1} = \frac{3}{4} \times \frac{35}{1} = \frac{105}{4}$ or $26\frac{1}{4}$

R.1.13: - Find $2\frac{7}{8} \times \frac{2}{3} = \frac{\cancel{23}^1}{\cancel{8}_4} \times \frac{\cancel{2}^1}{3} = \frac{23}{12}$ (or) $1\frac{11}{12}$

Diagram illustrating the conversion of the improper fraction $\frac{105}{4}$ to the mixed number $26\frac{1}{4}$. The diagram shows the long division of 105 by 4, resulting in a quotient of 26 and a remainder of 1. The remainder 1 is then placed over the divisor 4 to form the fractional part $\frac{1}{4}$.

Division of Fractions

Ex: - Find a) $\frac{3}{7} \div \frac{1}{7} = \frac{3}{7} \times \frac{7}{1} = \frac{3}{1} = 3$

b) $\frac{5}{8} \div \frac{7}{12} = \frac{5}{8} \times \frac{12}{7} = \frac{5}{2} \times \frac{3}{7} = \frac{15}{14}$

R.1.14: - Find $\frac{6}{25} \div \frac{2}{5} = \frac{\overset{3}{\cancel{6}}}{\underset{5}{\cancel{25}}} \times \frac{\overset{1}{\cancel{5}}}{\underset{1}{\cancel{2}}} = \frac{3}{5}$

R.1.15: - Evaluate

a) $1\frac{1}{3} \div \frac{8}{3} = \frac{4}{3} \times \frac{3}{8} = \frac{1}{2}$

b) $\frac{20}{21} \div \frac{5}{7} = \frac{20}{21} \times \frac{7}{5} = \frac{4}{3}$ (or) $1\frac{1}{3}$

For Extra Questions & Students Queries

R3:- Percentage and Ratio

Percentage: - Percent means per hundred. Percentage is a fraction of 100. It is often denoted by using the symbol “%”.

$$\text{Ex: - } 10\% = \frac{10}{100} = \frac{1}{10}, \quad 5\% = \frac{5}{100} = \frac{1}{20}$$

Ratio: - It is the quantitative relation between two numbers, indicating how many times the first number contains or contained in the second number.

Percentages $\Rightarrow$ Fractions $\Rightarrow$ Decimals

To convert percentages into fractions and then into decimals, divide the given percentage by 100 and remove the % symbol.

Examples: Convert the following percentages into fractions and then into decimals.

1. $12\% = \frac{12}{100} = \frac{3}{25}$ (Fraction form);

$$12\% = \frac{12}{100} = 0.12 \quad (\text{Decimal form})$$

2. $17.5\% = \frac{17.5}{100} = \frac{175}{1000} = \frac{7}{40}$ (Fraction form);

$$17.5\% = \frac{17.5}{100} = 0.175 \quad (\text{Decimal form})$$

3. $28\% =$ _____ (Fraction form)

$$28\% =$$
 _____ (Decimal form)

Fractions or Decimals $\Rightarrow$ Percentages

To convert fractions or decimals into percentages, multiply the given fraction or decimal with 100 and postfix the % symbol.

Examples: Convert the following fractions of decimals into percentages.

1. $\frac{3}{5} = \frac{3}{5} \times 100\% = 60\%$

2. $\frac{7}{15} = \frac{7}{15} \times 100\% = \frac{7}{3} \times 20\% = \frac{140}{3}\% = 46\frac{2}{3}\%$

3. $\frac{8}{25} = \frac{8}{25} \times 100\% = 8 \times 4\% = 32\%$

4. $0.5 = 0.5 \times 100\% = 50\%$

5. $12.8 = 12.8 \times 100\% = 1280\%$

6. $2.036 = 2.03 \times 100\% = 203.6\%$

❖ **Certain percentage of a given quantity: -**

Rule: - $x\%$ of a quantity $y = \frac{x}{100} * y$

Examples:

1. Find 40 % of 240

Sol: $40\% \text{ of } 240 = \frac{40}{100} \times 240 = 96$

2. Find 15 % of \$250.

Sol: $15\% \text{ of } \$250 = \frac{15}{100} \times 250 = \37.5

3. In a public show, 75 % of the seats were filled. If there were 600 seats in the hall, how many seats were vacant?

Sol: **First process:**

Number of seats filled = $75\% \text{ of } 600 = \frac{75}{100} \times 600 = 450$

Therefore, the number of vacant seats = $600 - 450 = 150$.

Second process:

Total percentage of seats = 100%.

Percentage of filled seats = 75%.

Therefore, percentage of vacant seats = $100 - 75 = 25\%$.

$$25 \% \text{ of } 600 = \frac{25}{100} \times 600 = 150$$

Thus, the number of vacant seats = 150.

4. The cost of a book is SR 200. What is the price to be paid if there is a discount of 10% on its cost?

$$90 \% \text{ of } 200 = \frac{90}{100} \times 200 = 180$$

5. A deposit of £ 750 increases by 9%. Calculate the resulting deposit.

$$9 \% \text{ of } 750 = \frac{9}{100} \times 750 = \text{£ } 67.50$$

$$\text{The resulting deposit} = 750 + 67.50 = \text{£ } 817.50$$

❖ Finding Value of quantity when certain percentage of it is given

Rule: - If $x\%$ of a quantity $m = y$, then $m = ?$

$$\frac{x}{100} * m = y \Rightarrow m = \frac{y}{x} * 100$$

Thus, the value of quantity $m = \frac{y}{x} * 100$

1. Find x if: 9% of x is \$36

$$\text{Sol: } \frac{9}{100} * x = 36 \Rightarrow x = \frac{36}{9} * 100 = 4 * 100 = \$400.$$

2. Ex: - What is the sum of the money of which 15% is \$225?

Sol: Let the required sum of money be \$ m .

$$15\% \text{ of } \$m = \$225 \quad \Rightarrow \frac{15}{100} \times m = 225$$

$$\Rightarrow m = \frac{225 \times 100}{15} \quad \Rightarrow m = 1500$$

Therefore, sum of the money = \$1500.

❖ Percentage of one quantity out of another quantity

Rule: - The percentage of x out of quantity y is $\frac{x}{y} * 100 \%$

Examples:

1. Mohammed scored 36 marks out of 60. Express the marks in percentage.

Solution: Required percent = $(\frac{36}{60} \times 100) \% = 60\%$

2. Express 80 ml as a percent of 400 ml

Solution: Required percent = $(\frac{80}{400} \times 100) \% = 20 \%$

3. What percent of \$15 is 75 cents?

Solution: We know, \$1 = 100 cents

\$15 = (15 × 100) cents = 1500 cents

Therefore, required percent = $(\frac{75}{1500} \times 100) \% = 5 \%$

4. What percent of 70 kg is 2.1 kg?

Solution: Required percent = $\left(\frac{2.1}{70} \times 100\right) \% = \frac{210}{70} \% = 3 \%$

Therefore, 2.1 kg is 3 % of 70 kg.

Percentage change:

$$\text{Percentage change} = \frac{\text{change}}{\text{original value}} \times 100\% = \frac{\text{new value} - \text{original value}}{\text{original value}} \times 100\%$$

1. A worker's earnings increase from £ 10000 to £ 11500. Calculate the percentage change.

$$\begin{aligned}\text{Percentage change} &= \frac{\text{new value} - \text{original value}}{\text{original value}} \times 100\% \\ &= \frac{11500 - 10000}{10000} \times 100\% = \frac{1500}{10000} \times 100\% = 15\%\end{aligned}$$

2. A microwave oven is reduced in price from £ 150 to £ 135. Calculate the percentage change in price.

$$\text{Percentage change} = \frac{\text{new value} - \text{original value}}{\text{original value}} \times 100\%$$

$$= \frac{135 - 150}{150} \times 100\% = \frac{-15}{150} \times 100\% = -10\%$$

The negative result is indicative of the price decrease. The percentage change in price is -10%.

Ratio

Examples

1. Divide 170 in the ratio 3 : 2

Total number of parts = 3 + 2 = 5

$$\text{First value} = \frac{3}{5} \times 170 = 102$$

$$\text{Second value} = \frac{2}{5} \times 170 = 68$$

Percentage into Ratio

Step I: Obtain the percentage.

Step II: Convert the given percentage into fraction by dividing it by 100 and removing percentage symbol (%).

Step III: Reduce the fraction obtained in **step II** in the simplest form.

Step IV: Write the fraction obtained in **step III** as a ratio.

1. Express each of the following percentage as ratios in the simplest form:

(i) $46\% = 46/100 = 23/50 = 23 : 50$

(ii) $20\% = 20/100 = 1/5 = 1 : 5$

(v) $1\% = 1/100 = 1 : 100$

Ratio into Percentage

Step I: Obtain the ratio. Let the ratio be $x : y$

Step II: Convert the given ratio into the fraction x/y .

Step III: Multiply the fraction obtained in **step II** by 100 and put the percentage sign(%).

1. Express each of the following ratios into percentage:

(i) $6 : 5 = 6/5 = (6/5 \times 100) \% = 600/5 \% = 120 \%$

(ii) $8 : 25 = 8/25 = (8/25 \times 100) \% = 32 \%$

(iii) $10 : 50 = 10/50 = (1/5 \times 100) \% = 20 \%$

(iii) $5 : 8 = 5/8 = (5/8 \times 100) \% = 500/8 \% = 125/2 \% = 62.5 \%$

For Extra Questions & Students Queries

Measurements

Measurement of Lengths: - The most common measurements of lengths are:

- **Millimeters (mm)** ($1 \text{ mm} = 1/10 \text{ cm}$ and $1 \text{ mm} = 1/1000 \text{ m}$)
- **Centimeters (cm)** ($1 \text{ cm} = 10 \text{ mm}$; $1 \text{ cm} = 1/100 \text{ m}$)
- **Meters (m)** ($1 \text{ m} = 100 \text{ cm} = 1000 \text{ mm}$)
- **Kilometers (km)** ($1 \text{ km} = 1000 \text{ m}$)

Metric system (**SI**) of unit of length is **meter**.

➤ Other measurements of lengths

- **Inches** ($1 \text{ inch} = 2.54 \text{ cm}$)
- **Foot** ($1 \text{ ft} = 12 \text{ inches}$)
- **Yard** ($1 \text{ yard} = 3 \text{ ft}$; $1 \text{ yard} = 0.91 \text{ m}$)

➤ Measurements of longer distances

- **Miles** ($1 \text{ Mile} = 1.609 \text{ km}$; $1 \text{ km} = 0.621 \text{ miles}$)

Measurement of Mass: - The most common measurements of mass are:

- **Milligrams (mg)** (1 mg = 1/1000 g)
- **Grams (g)** (1 g = 1000 mg; 1 g = 1/1000 kg)
- **Kilograms (kg)** (1 kg = 1000 g)
- **Tonne** (1 ton = 1000 kg)

Metric system (**SI**) of unit for mass is **Kilogram (Kg)**.

Measurement of Liquid Volume: - The most common measurements of liquids are:

- **Milliliters (ml)** (1 ml = 1/1000 lit.)
- **Gallon** (1 US gallon = 3.785 l)
- **Liters (l)** (1 lit = 1000 ml)
- **Kiloliter (kl)** (1 kl = 1000 l)

Examples

1. Fill in the following blanks:

a) $2.8 \text{ km} = \underline{2800} \text{ m.}$ b) $150 \text{ cm} = \underline{1.5} \text{ m.}$

c) $5 \text{ km} = \underline{3.105} \text{ miles.}$

2. Fill in the following blanks.

a) $2.5 \text{ kg} = \underline{2500} \text{ g.}$

b) $500 \text{ g} = \underline{0.5} \text{ kg.}$

3. Fill in the following blanks.

a) $35 \text{ L} = \underline{35000} \text{ ml.}$

b) $100 \text{ ml} = \underline{0.1} \text{ L.}$

For Extra Questions & Students Queries