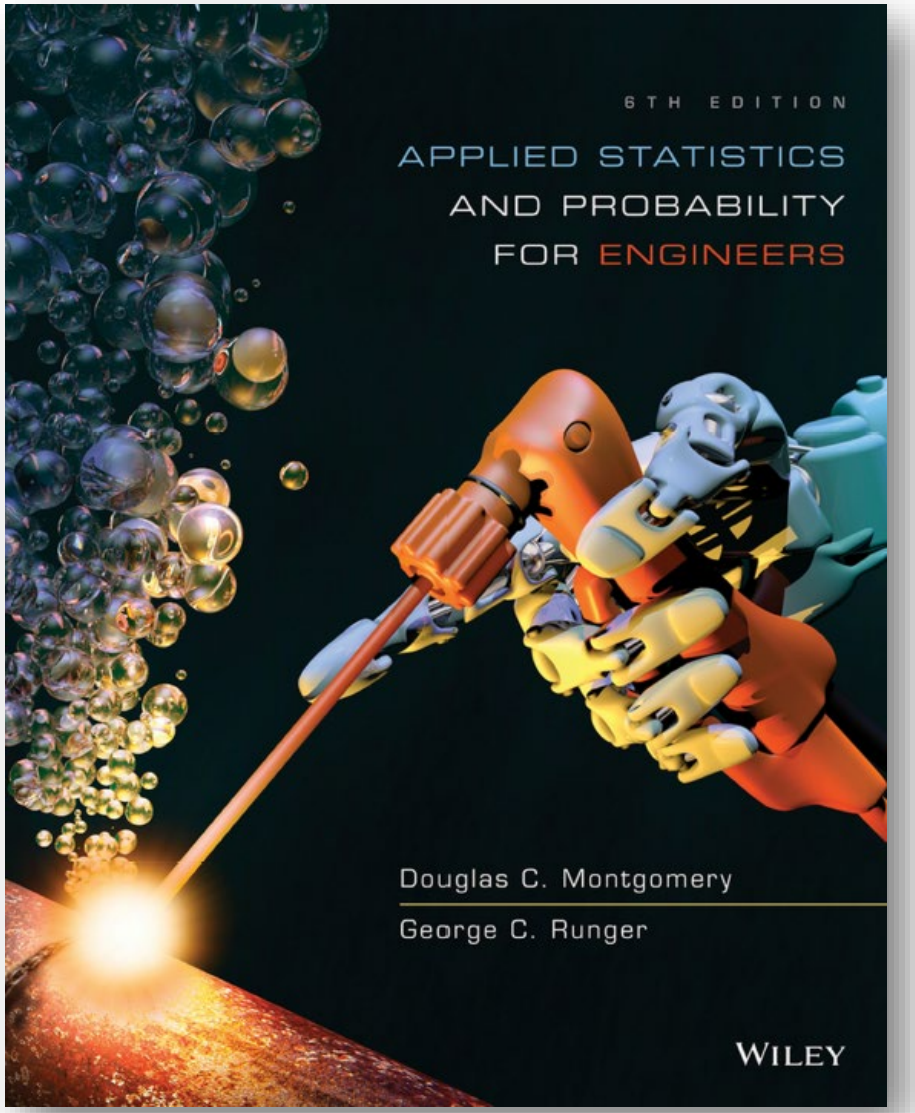

Probability and Statistics

King Faisal University

College of Science

Department of Mathematics and Statistics

Math 121

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Sample Spaces and Events

Random Experiment

An experiment that can result in different outcomes, even though it is repeated in the same manner every time, is called a **random experiment**.

Examples:

For the example of measuring current in a copper wire, our model for the system might simply be Ohm's law. Because of uncontrollable inputs, variations in measurements of current are expected. Ohm's Law might be a suitable approximation.

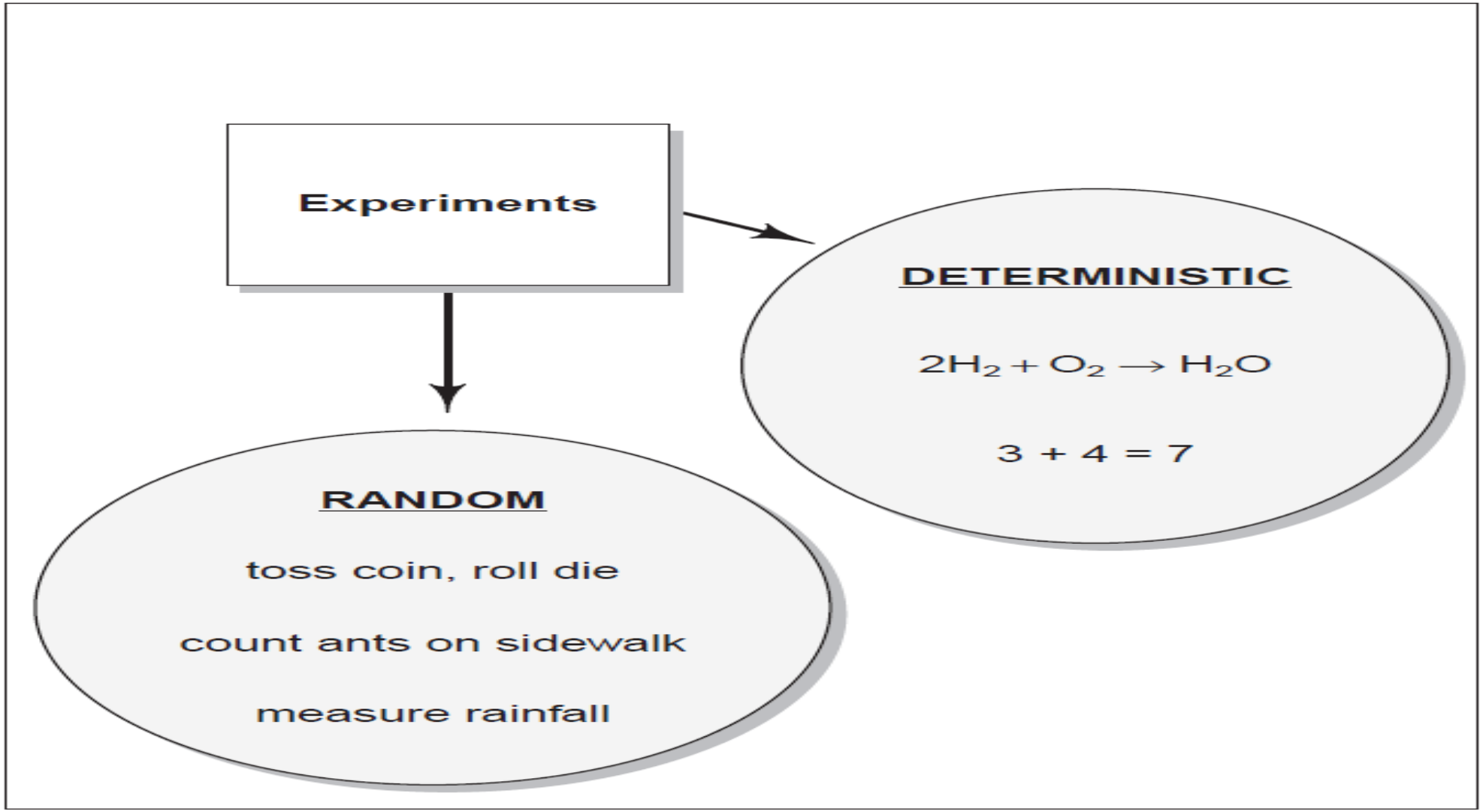


Figure 4.1: Two types of experiments.

Sample Space

The set of all possible outcomes of a random experiment is called the **sample space** of the experiment. The sample space is denoted as S.

Example 2-1 **Camera Flash** Consider an experiment that selects a cell phone camera and records the recycle time of a flash (the time taken to ready the camera for another flash). The possible values for this time depend on the resolution of the timer and on the minimum and maximum recycle times. However, because the time is positive it is convenient to define the sample space as simply the positive real line

$$S = R^+ = \{x \mid x > 0\}$$

If it is known that all recycle times are between 1.5 and 5 seconds, the sample space can be

$$S = \{x \mid 1.5 < x < 5\}$$

If the objective of the analysis is to consider only whether the recycle time is low, medium, or high, the sample space can be taken to be the set of three outcomes

$$S = \{low, medium, high\}$$

If the objective is only to evaluate whether or not a particular camera conforms to a minimum recycle time specification, the sample space can be simplified to a set of two outcomes

$$S = \{yes, no\}$$

that indicates whether or not the camera conforms.

Discrete and Continuous Sample Spaces

Discrete and Continuous Sample Spaces

A sample space is **discrete** if it consists of a finite or countable infinite set of outcomes.

A sample space is **continuous** if it contains an interval (either finite or infinite) of real numbers.

Example 2-2 **Camera Specifications** Suppose that the recycle times of two cameras are recorded. The extension of the positive real line R is to take the sample space to be the positive quadrant of the plane

$$S = R^+ \times R^+$$

If the objective of the analysis is to consider only whether or not the cameras conform to the manufacturing specifications, either camera may or may not conform. We abbreviate *yes* and *no* as y and n. If the ordered pair yn indicates that the first camera conforms and the second does not, the sample space can be represented by the four outcomes:

$$S = \{yy, yn, ny, nn\}$$

If we are interested only in the number of conforming cameras in the sample, we might summarize the sample space as

$$S = \{0, 1, 2\}$$

As another example, consider an experiment in which cameras are tested unitl the flash recycle time fails to meet the specifications. The sample space can be represented as

$$S = \{n, yn, yyn, yyyn, yyyyn, \text{and so forth}\}$$

and this is an example of a discrete sample space that is countably infinite.

Tree Diagrams

- Sample spaces can also be described graphically with tree diagrams. When a sample space can be constructed in several steps or stages.
- we can represent each of the $n1$ ways of completing the first step as a branch of a tree. Each of the ways of completing the second step can be represented as $n2$ branches starting from the ends of the original branches, and so forth.

Examples:

Example 2-3 **Message Delays** Each message in a digital communication system is classified as to whether it is received within the time specified by the system design. If three messages are classified, use a tree diagram to represent the sample space of possible outcomes.

Each message can be received either on time or late. The possible results for three messages can be displayed by eight branches in the tree diagram shown in Fig. 2-5.

Practical Interpretation: A tree diagram can effectively represent a sample space. Even if a tree becomes too large to construct, it can still conceptually clarify the sample space.

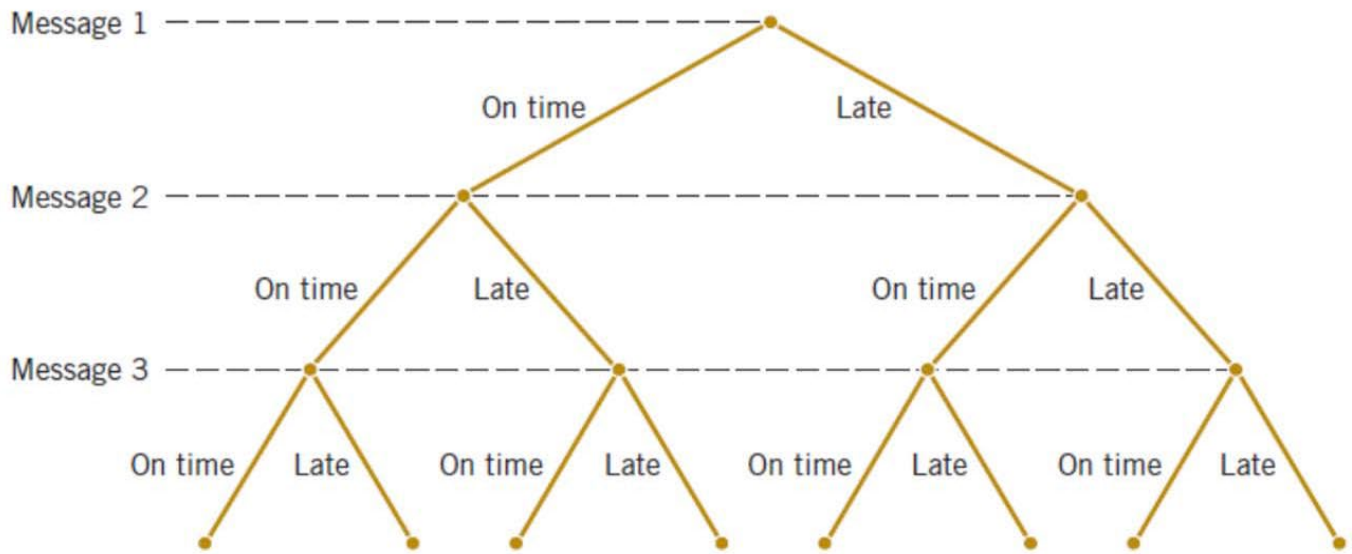


FIGURE 2-5 Tree diagram for three messages.

Examples:

Example 2-4 **Automobile Options** An automobile manufacturer provides vehicles equipped with selected options. Each vehicle is ordered

- With or without an automatic transmission
- With or without a sunroof
- With one of three choices of a stereo system
- With one of four exterior colors

If the sample space consists of the set of all possible vehicle types, what is the number of outcomes in the sample space?

The sample space contains 48 outcomes. The tree diagram for the different types of vehicles is displayed in Fig. 2-6.

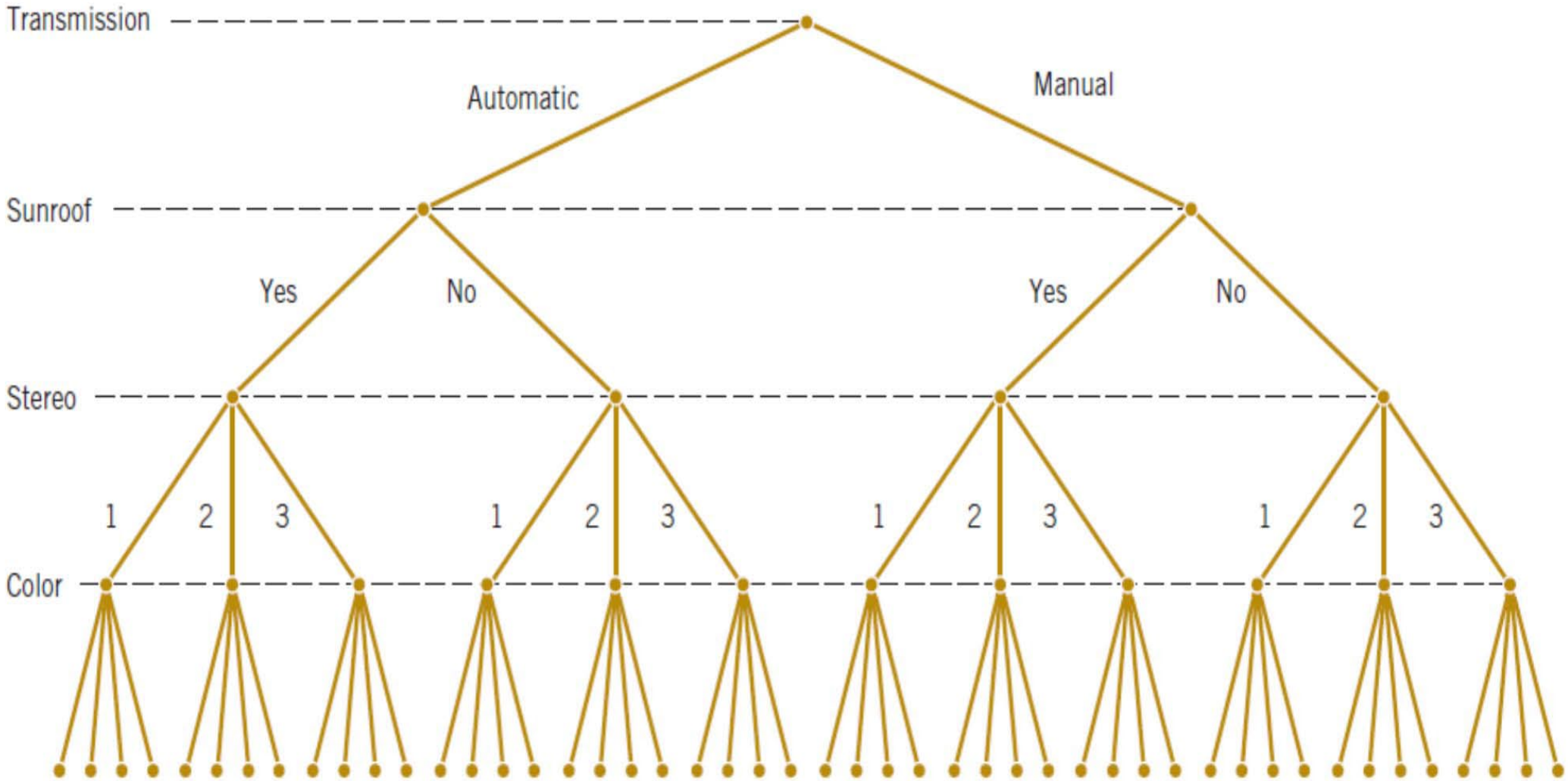


FIGURE 2-6 Tree diagram for different types of vehicles with 48 outcomes in the sample space.

Events

Event

An **event** is a subset of the sample space of a random experiment.

Example:

Consider an experiment in which each of three vehicles taking a particular freeway exit turns left (L) or right (R) at the end of the exit ramp. The eight possible outcomes that comprise the sample space are LLL, RLL, LRL, LLR, LRR, RLR, RRL, and RRR. Thus there are eight simple events, among which are $E_1 = \{LLL\}$ and $E_5 = \{LRR\}$. Some compound events include

$A = \{RLL, LRL, LLR\}$ = the event that exactly one of the three vehicles turns right

$B = \{LLL, RLL, LRL, LLR\}$ = the event that at most one of the vehicles turns right

$C = \{LLL, RRR\}$ = the event that all three vehicles turn in the same direction

Suppose that when the experiment is performed, the outcome is LLL. Then the simple event E_1 has occurred and so also have the events B and C (but not A). ■

Example

When the number of pumps in use at each of two six-pump gas stations is observed, there are 49 possible outcomes, so there are 49 simple events: $E_1 = \{(0, 0)\}, E_2 = \{(0, 1)\}, \ldots, E_{49} = \{(6, 6)\}$. Examples of compound events are

$A = \{(0, 0), (1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)\}$ = the event that the number of pumps in use is the same for both stations

$B = \{(0, 4), (1, 3), (2, 2), (3, 1), (4, 0)\}$ = the event that the total number of pumps in use is four

$C = \{(0, 0), (0, 1), (1, 0), (1, 1)\}$ = the event that at most one pump is in use at each station

Example

The sample space for the program compilation experiment contains an infinite number of outcomes, so there are an infinite number of simple events. Compound events include

$A = \{S, FS, FFS\}$ = the event that at most three programs are examined

$E = \{FS, FFFS, FFFFFS, \ldots\}$ = the event that an even number of programs are examined

Basic set operations

The **union** of two events is the event that consists of all outcomes that are contained in either of the two events.

We denote the union as: $E_1 \cup E_2$

The **intersection** of two events is the event that consists of all outcomes that are contained in both of the two events.

We denote the intersection as: $E_1 \cap E_2$

The **complement** of an event in a sample space is the set of outcomes in the sample space that are not in the event.

We denote the complement of the event E as E'.

The notation E^c is also used in other literature to denote the complement.

Example

Consider the following sample space:

$$S = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$$

And let:

E_1: set of all odd numbers = $\{1, 3, 5, 7, 9\}$

E_2: set of all even numbers = $\{2, 4, 6, 8, 10\}$

Then:

$E_1 \cup E_2 = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} = S$

$E_1 \cap E_2 = \emptyset$

${E_1}^c = \{2, 4, 6, 8, 10\} = E_2$

Example

Example 2-6 **Events** Consider the sample space $S = \{yy, yn, ny, nn\}$ in Example 2-2. Suppose that the subset of outcomes for which at least one camera conforms is denoted as E_1. Then,

$$E_1 = \{yy, yn, ny\}$$

The event such that both cameras do not conform, denoted as E_2, contains only the single outcome, $E_2 = \{nn\}$. Other examples of events are $E_3 = \emptyset$, the null set, and $E_4 = S$, the sample space. If $E_5 = \{yn, ny, nn\}$,

$$E_1 \cup E_5 = S \qquad E_1 \cap E_5 = \{yn, ny\} \qquad E_1' = \{nn\}$$

Practical Interpretation: Events are used to define outcomes of interest from a random experiment. One is often interested in the probabilities of specified events.

Example

Example 2-7

As in Example 2-1, camera recycle times might use the sample space $S = R^+$, the set of positive real numbers. Let

$$E_1 = \{x \mid 10 \le x < 12\} \quad \text{and} \quad E_2 = \{x \mid 11 < x < 15\}$$

Then,

$$E_1 \cup E_2 = \{x \mid 10 \le x < 15\}$$

and

$$E_1 \cap E_2 = \{x \mid 11 < x < 12\}$$

Also,

$$E_1' = \{x \mid x < 10 \quad \text{or} \quad 12 \le x\}$$

and

$$E_1' \cap E_2 = \{x \mid 12 \le x < 15\}$$

Mutually Exclusive Events

Mutually Exclusive Events

Two events, denoted as E_1 and E_2, such that

$$E_1 \cap E_2 = \emptyset$$

are said to be **mutually exclusive**.

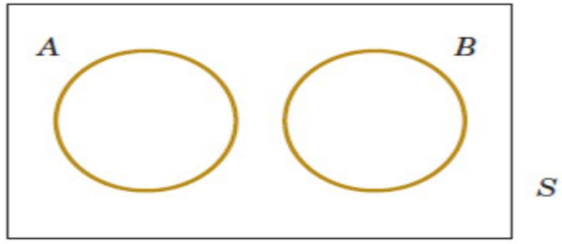


FIGURE 2-9 Mutually exclusive events.

Additional results involving events are summarized in the following. The definition of the complement of an event implies that

$$(E')' = E$$

The distributive law for set operations implies that

$$(A \cup B) \cap C = (A \cap C) \cup (B \cap C) \quad \text{and} \quad (A \cap B) \cup C = (A \cup C) \cap (B \cup C)$$

DeMorgan's laws imply that

$$(A \cup B)' = A' \cap B' \quad \text{and} \quad (A \cap B)' = A' \cup B'$$

Also, remember that

$$A \cap B = B \cap A \quad \text{and} \quad A \cup B = B \cup A$$

Venn diagrams

Diagrams are often used to portray relationships between sets, and these diagrams are also used to describe relationships between events. We can use Venn diagrams to represent a sample space and events in a sample space.

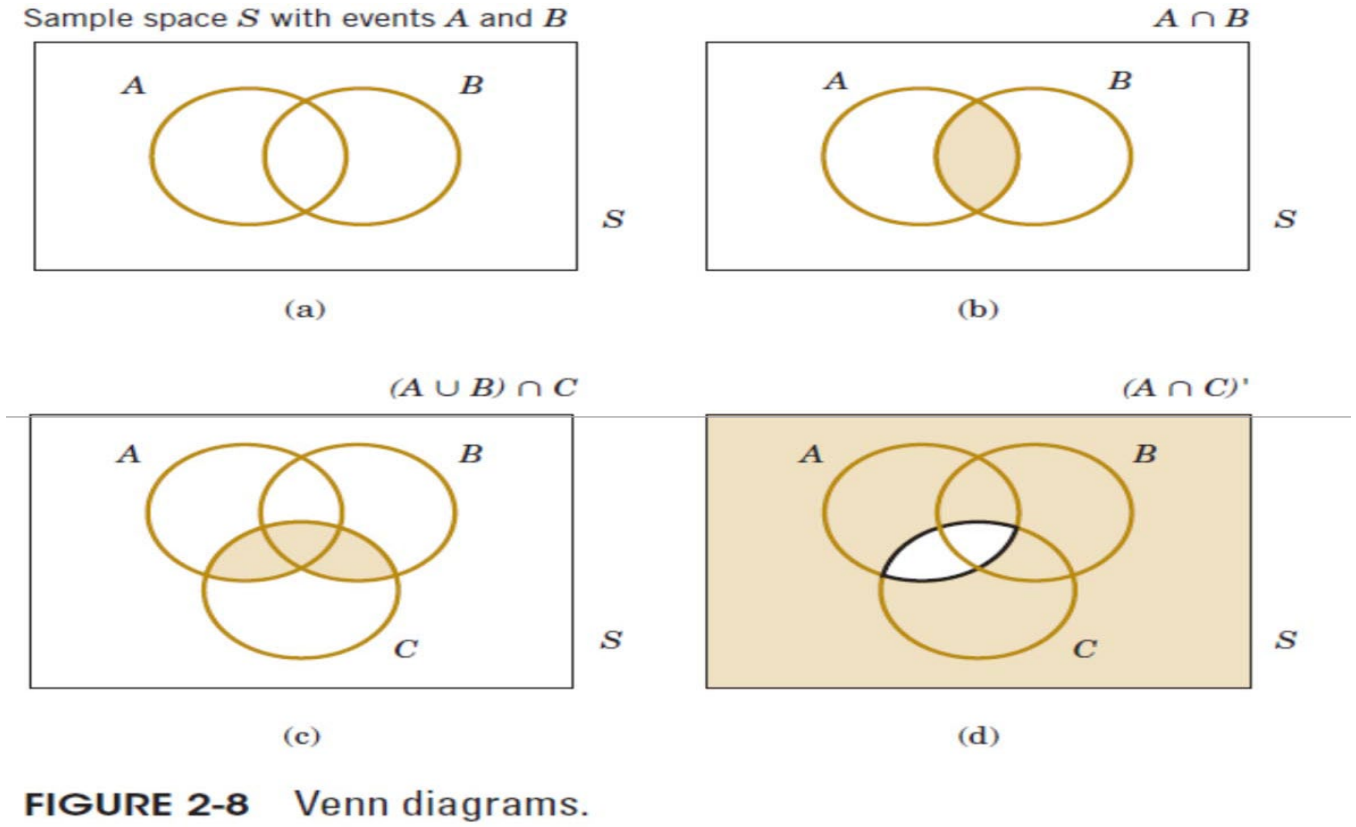
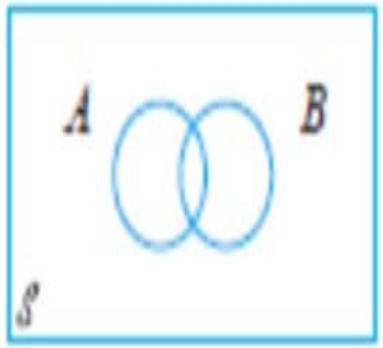
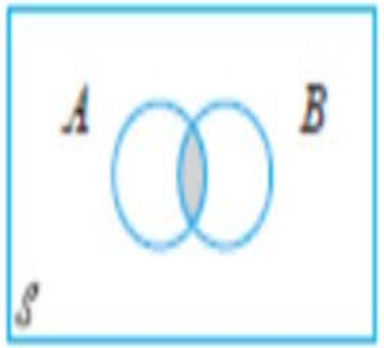


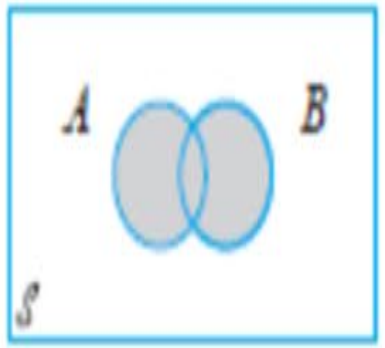
FIGURE 2-8 Venn diagrams.



(a) Venn diagram of events A and B

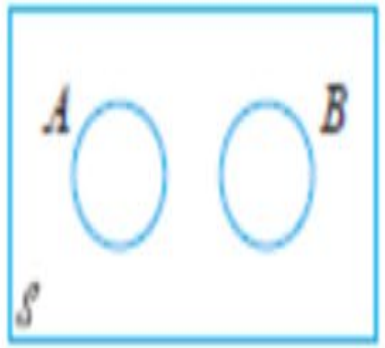


(b) Shaded region is $A \cap B$



(c) Shaded region is $A \cup B$

(d) Shaded region is A'

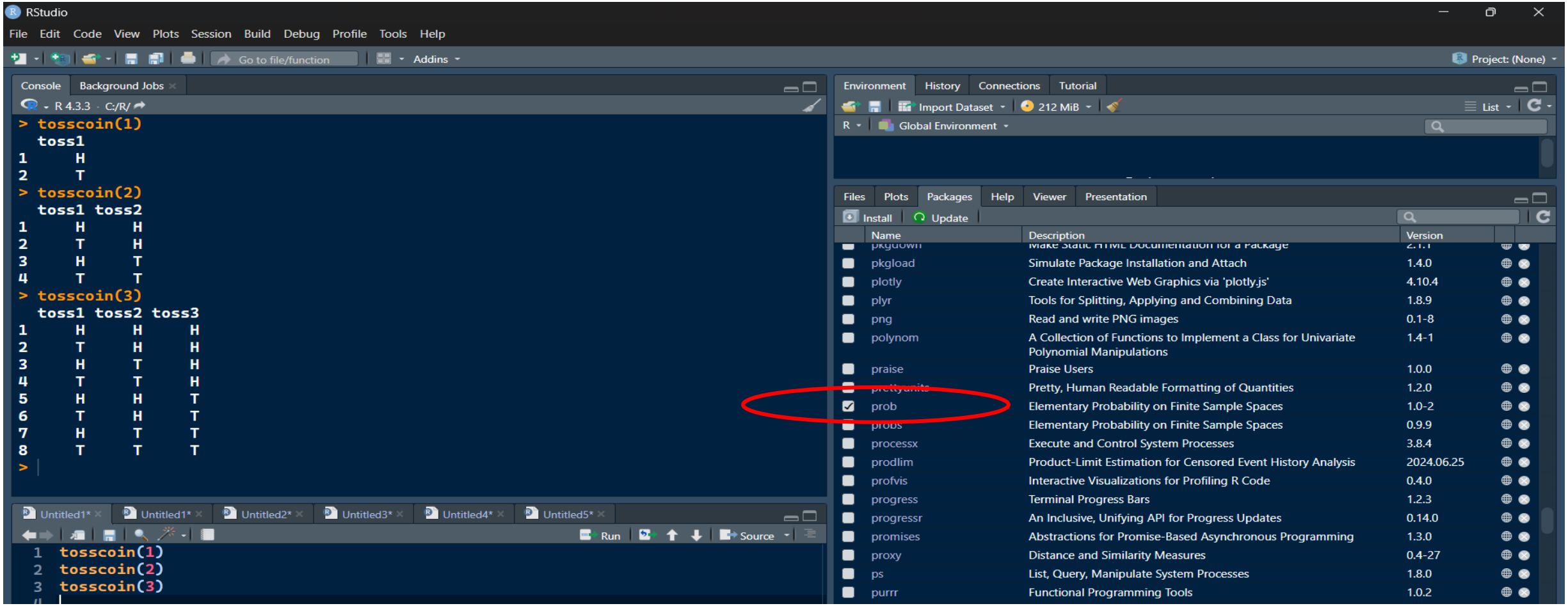


(e) Mutually exclusive events

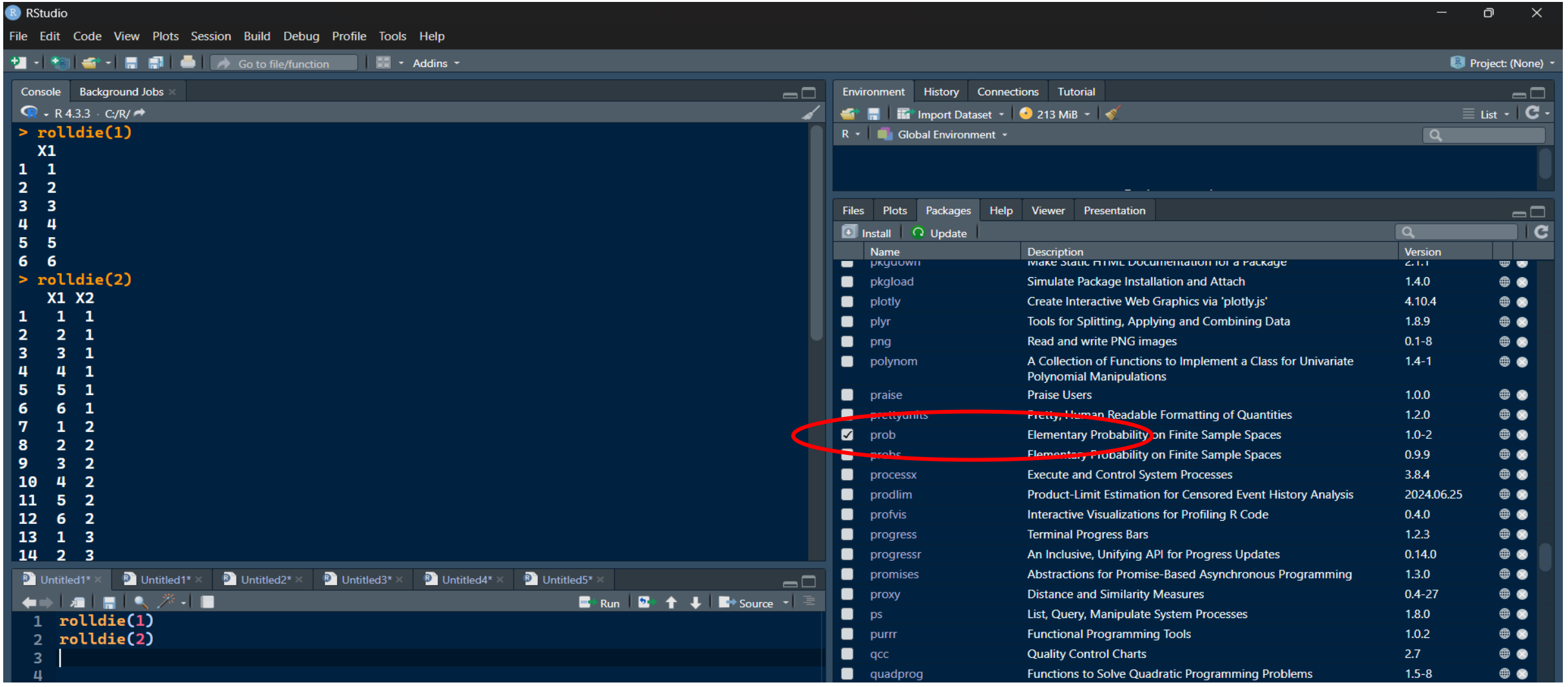
How to do it with R

Most of the probability work in this book is done with the *prob package*

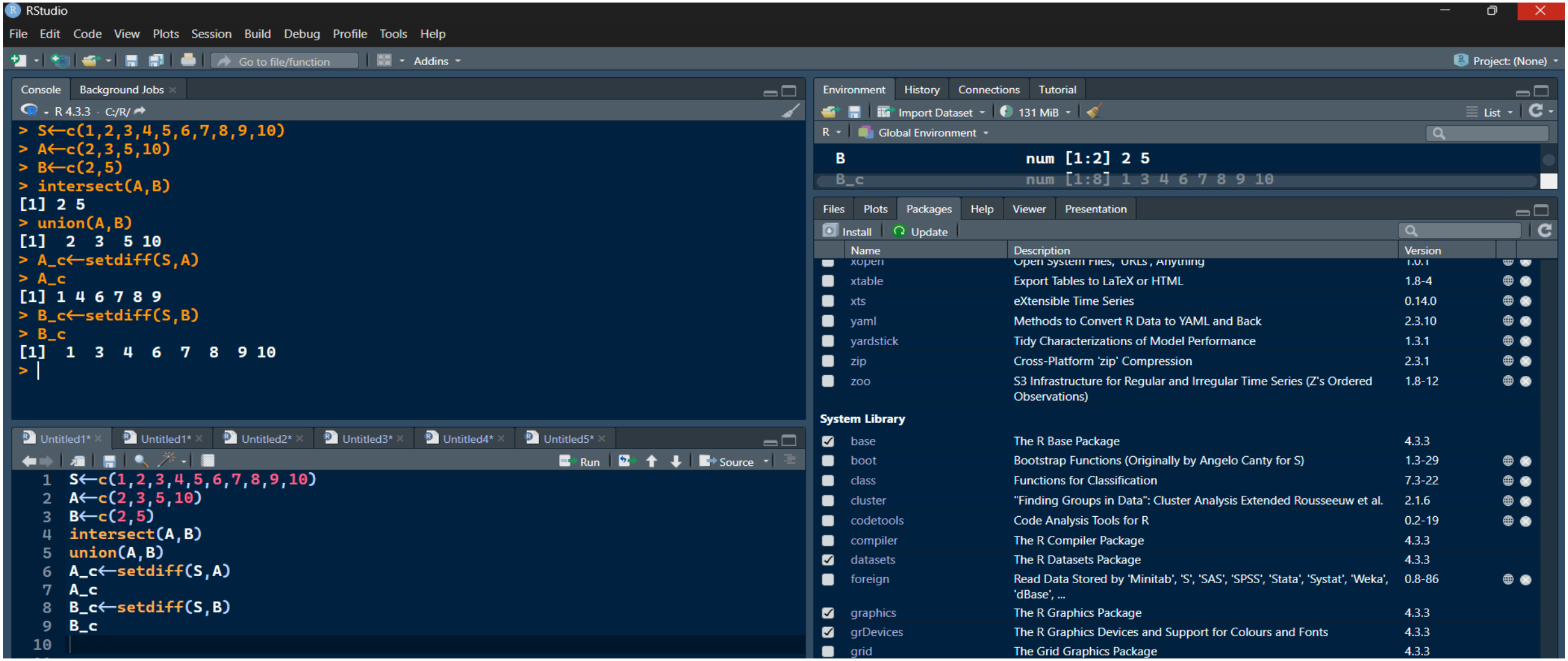
Toss one, two and three coins: (sample space)



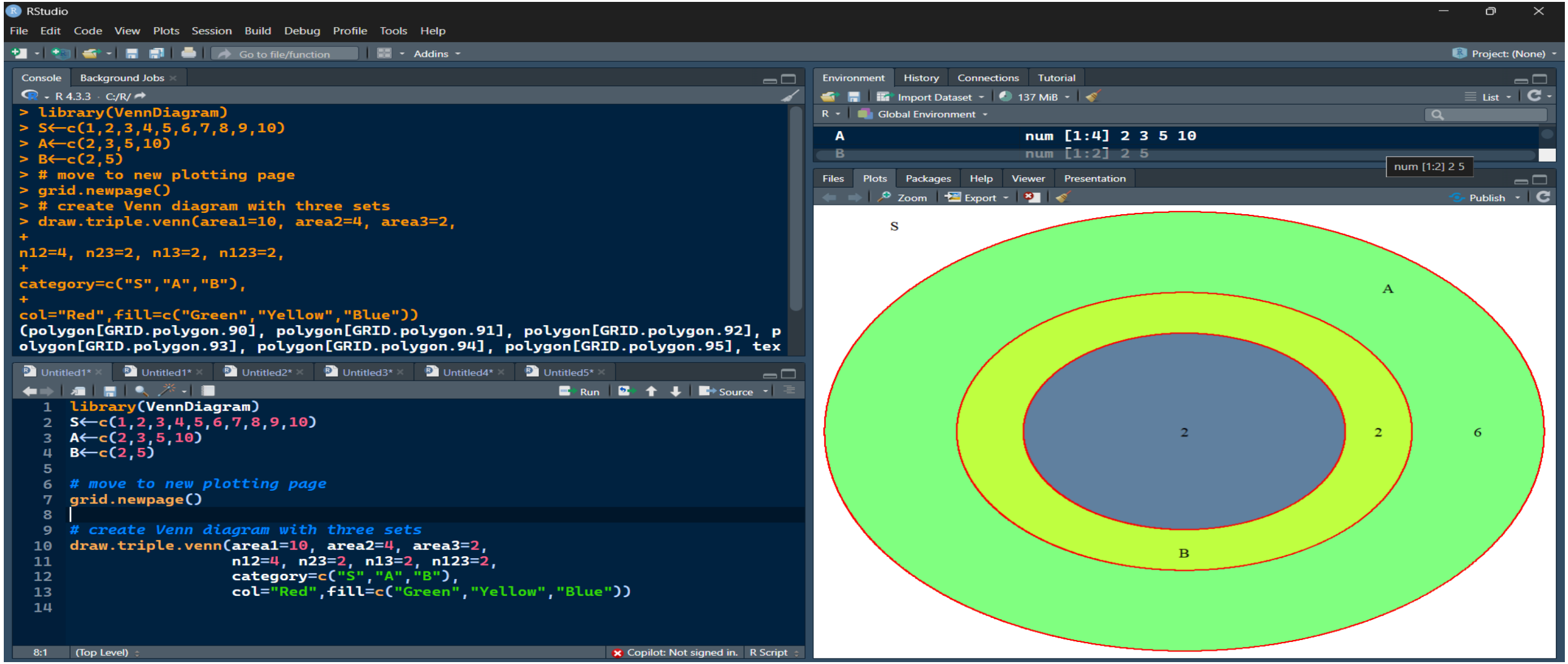
Roll one and two die:



Intersection, Union and Complement Sets with R:



Venn Diagram with R for 3 Sets:



The End

Thank you