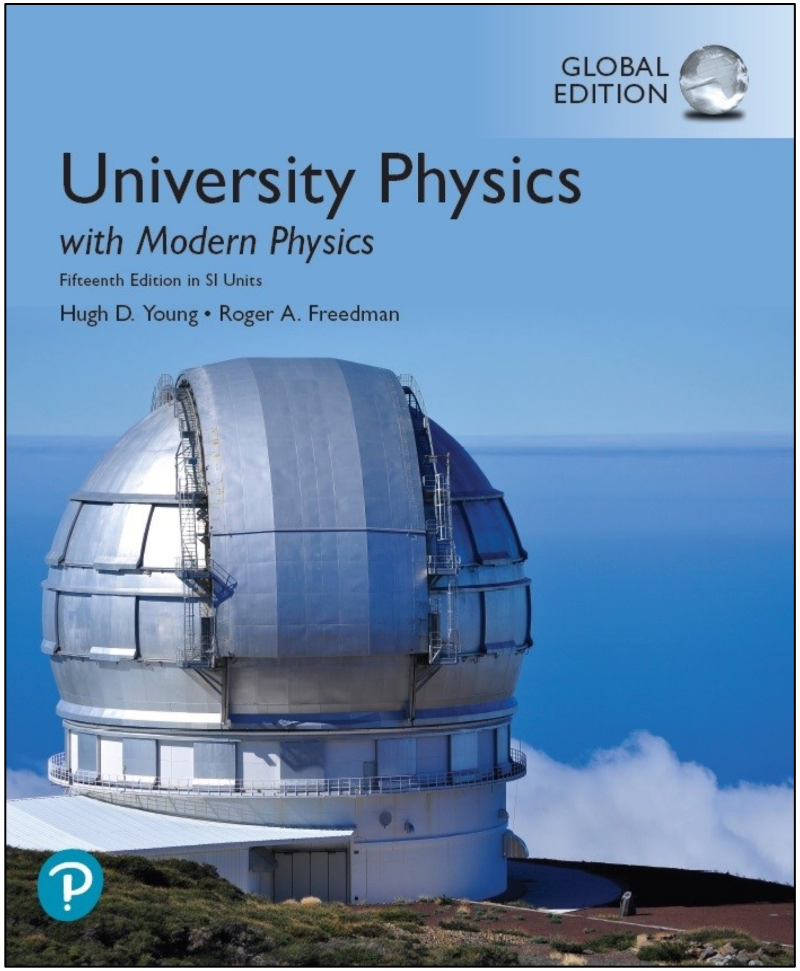
University Physics with Modern Physics

Fifteenth Edition, Global Edition, in SI Units



Chapter 1

Units, Physical Quantities, and Vectors

Learning Outcomes

In this chapter, you'll learn…

- the four steps you can use to solve any physics problem.
- three fundamental quantities of physics and the units physicists use to measure them.
- how to work with units and significant figures in your calculations.
- how to add and subtract vectors graphically, and using vector components.
- two ways to multiply vectors: the scalar (dot) product and the vector (cross) product.

The Nature of Physics

- Physics is an **experimental** science in which physicists seek patterns that relate the phenomena of nature.
- The patterns are called **physical theories**.
- A very well established or widely used theory is called a **physical law** or **principle**.

According to legend, Galileo investigated falling objects by dropping them from the Leaning Tower of Pisa, Italy, ...

... and he studied pendulum motion by observing the swinging chandelier in the adjacent cathedral.

Solving Problems in Physics

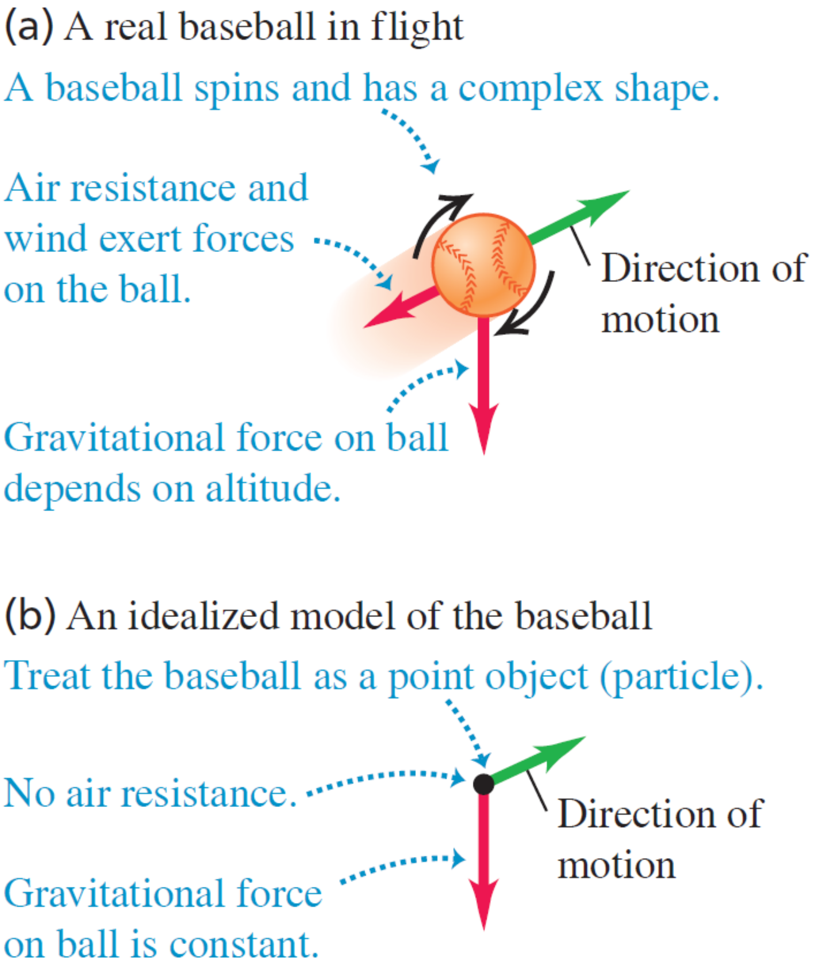
- All of the **Problem-Solving Strategies** and **Examples** in this book will follow these four steps:
- **Identify** the relevant concepts, target variables, and known quantities, as stated or implied in the problem.
- **Set Up** the problem: Choose the equations that you'll use to solve the problem, and draw a sketch of the situation.
- **Execute** the solution: This is where you "do the math."
- **Evaluate** your answer: Compare your answer with your estimates, and reconsider things if there's a discrepancy.

Idealized Models

To simplify the analysis of

a) a baseball in flight, we use

b) an idealized model.



Standards and Units

- Length, time, and mass are three **fundamental** quantities of physics.
- The **International System** (SI for **Système International**) is the most widely used system of units.
- In SI units, length is measured in **meters**, time in **seconds**, and mass in **kilograms**.

Unit Prefixes

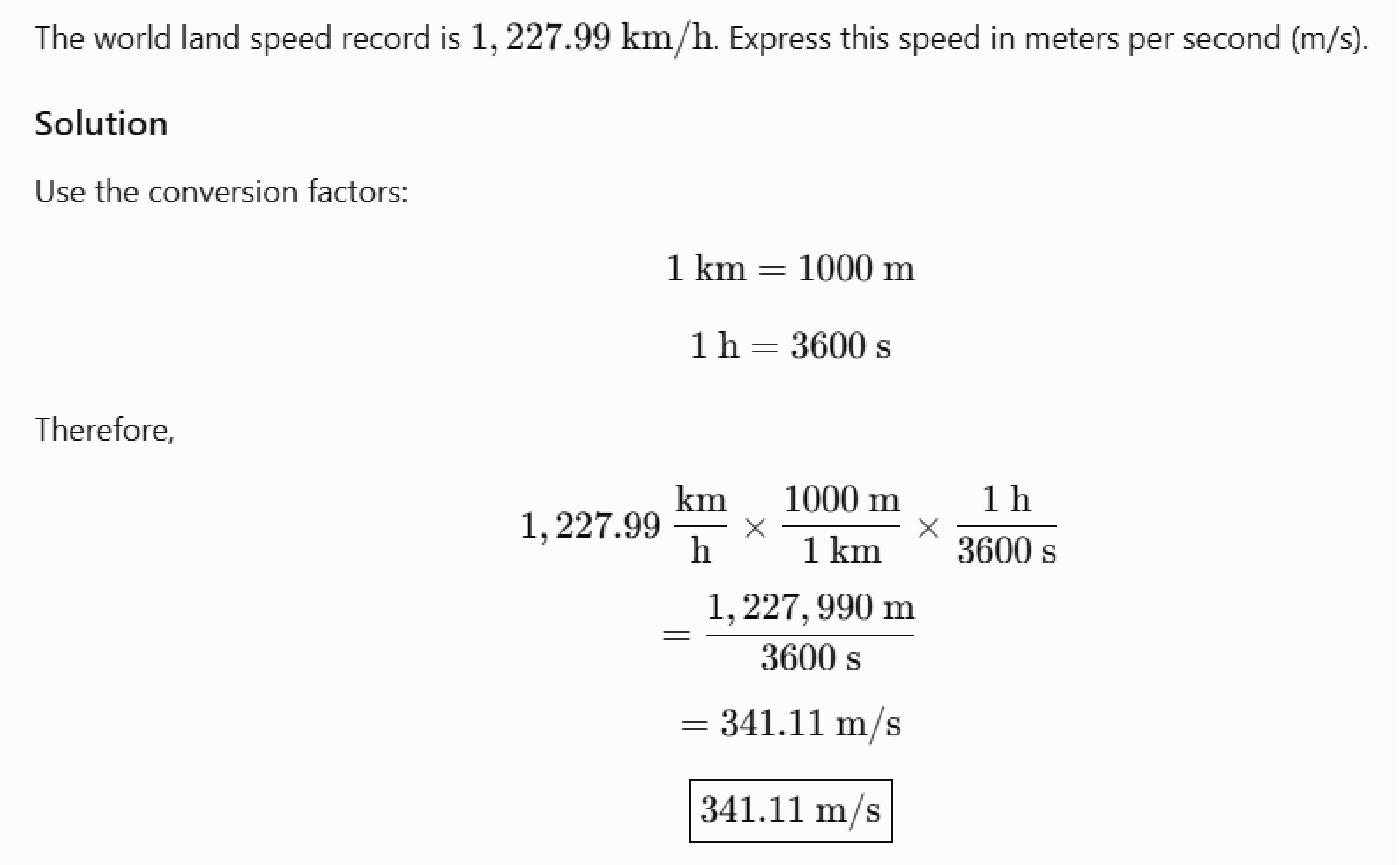
- Prefixes can be used to create larger and smaller units for the fundamental quantities. Some examples are:
- $1\ \mu\text{m} = 10^{-6}\ \text{m}$ (size of some bacteria and living cells)
- $1\ \text{km} = 10^{3}\ \text{m}$ (a 10-minute walk)
- $1\ \text{mg} = 10^{-6}\ \text{kg}$ (mass of a grain of salt)
- $1\ \text{g} = 10^{-3}\ \text{kg}$ (mass of a paper clip)
- $1\ \text{ns} = 10^{-9}\ \text{s}$ (time for light to travel 0.3 m)

Unit Consistency and Conversions

- An equation must be **dimensionally consistent**. Terms to be added or equated must **always** have the same units. (Be sure you're adding "apples to apples.")
- Always carry units through calculations.
- Convert to standard units as necessary, by forming a ratio of the same physical quantity in two different units, and using it as a multiplier.
- For example, to find the number of seconds in 3 min, we write:

$$3\text{ min} = (3\ \cancel{\text{min}})\left(\frac{60\text{ s}}{1\ \cancel{\text{min}}}\right) = 180\text{ s}$$

Question 1



The world land speed record is $1,227.99 \text{ km/h}$. Express this speed in meters per second (m/s).

Solution

Use the conversion factors:

$$1 \text{ km} = 1000 \text{ m}$$

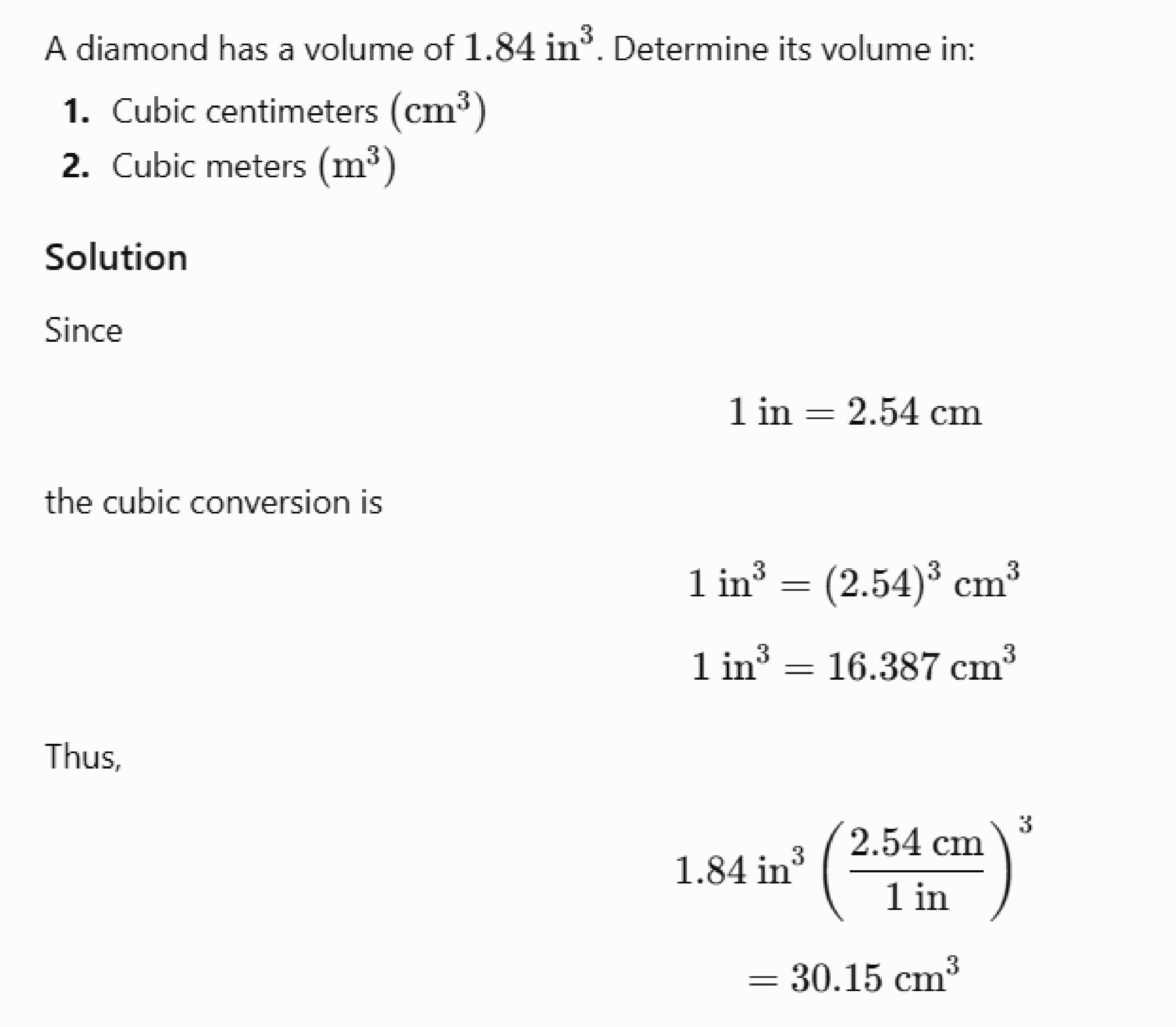
$$1 \text{ h} = 3600 \text{ s}$$

Therefore,

$$\begin{aligned} 1,227.99 \frac{\text{km}}{\text{h}} \times \frac{1000 \text{ m}}{1 \text{ km}} \times \frac{1 \text{ h}}{3600 \text{ s}} \\ = \frac{1,227,990 \text{ m}}{3600 \text{ s}} \\ = 341.11 \text{ m/s} \end{aligned}$$

$$\boxed{341.11 \text{ m/s}}$$

Question 2



A diamond has a volume of $1.84\ \mathrm{in}^3$. Determine its volume in:

1. Cubic centimeters (cm^3)
2. Cubic meters (m^3)

Solution

Since

$$1\ \mathrm{in} = 2.54\ \mathrm{cm}$$

the cubic conversion is

$$1\ \mathrm{in}^3 = (2.54)^3\ \mathrm{cm}^3$$

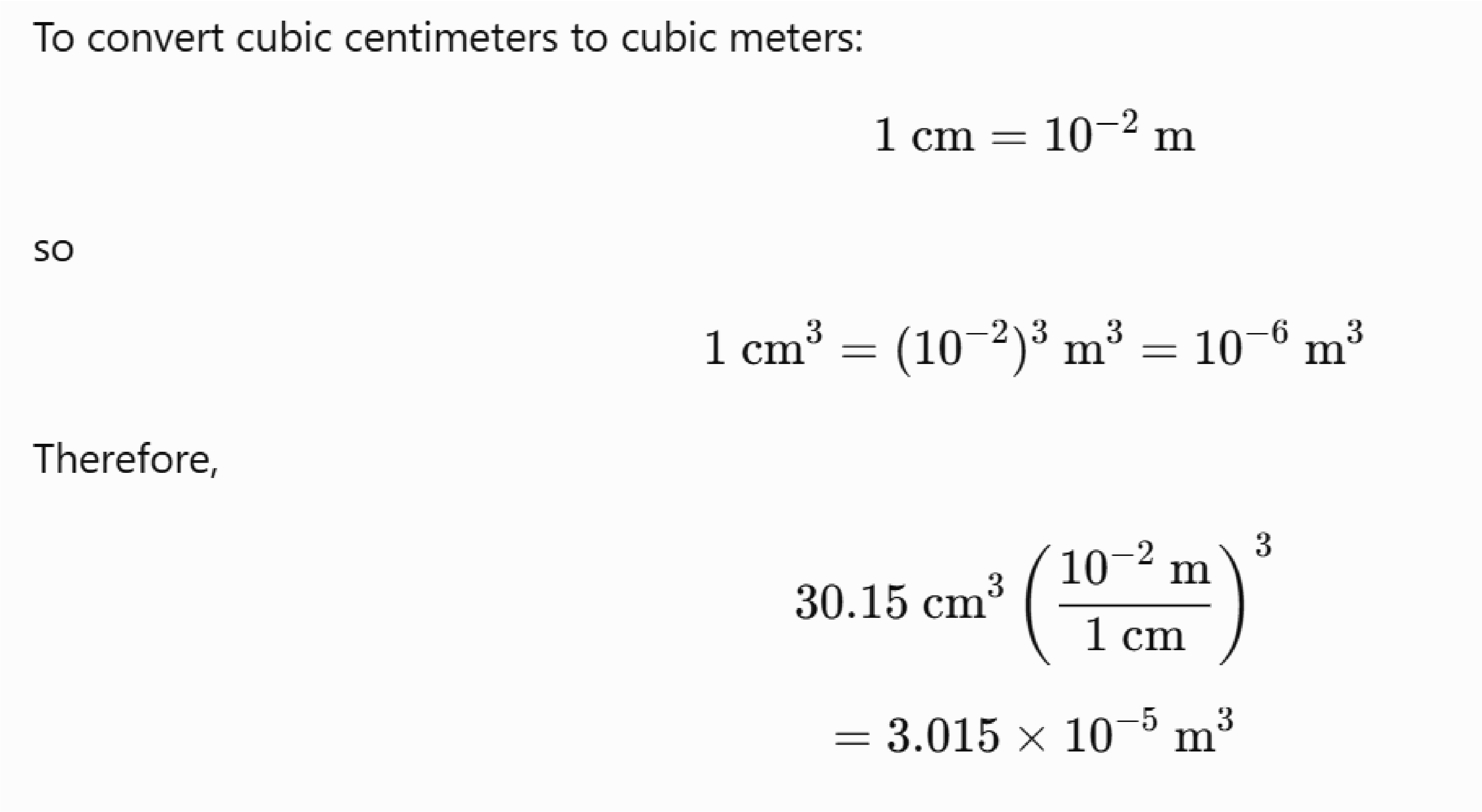
$$1\ \mathrm{in}^3 = 16.387\ \mathrm{cm}^3$$

Thus,

$$1.84\ \mathrm{in}^3 \left(\frac{2.54\ \mathrm{cm}}{1\ \mathrm{in}}\right)^3$$

$$= 30.15\ \mathrm{cm}^3$$

Continue….



To convert cubic centimeters to cubic meters:

$$1\ \text{cm} = 10^{-2}\ \text{m}$$

so

$$1\ \text{cm}^3 = (10^{-2})^3\ \text{m}^3 = 10^{-6}\ \text{m}^3$$

Therefore,

$$30.15\ \text{cm}^3 \left(\frac{10^{-2}\ \text{m}}{1\ \text{cm}}\right)^3$$

$$= 3.015 \times 10^{-5}\ \text{m}^3$$

Uncertainty and Significant Figures

- The uncertainty of a measured quantity is indicated by its number of **significant figures**.
- For multiplication and division, the answer can have no more significant figures than the **smallest** number of significant figures in the factors.
- For addition and subtraction, the number of significant figures is determined by the term having the fewest digits to the right of the decimal point.
- As this train mishap illustrates, even a small percent error can have spectacular results!
- Video Tutor Solution: Example 1.3

What do significant figures indicate about a measured quantity?

A. Its exact value
B. Its uncertainty
C. Its unit
D. Its conversion factor

Which rule is used when multiplying or dividing measured quantities?

A. Use the fewest decimal places.
B. Use the greatest number of significant figures.
C. Use the smallest number of significant figures.
D. Always use three significant figures.

Which rule is used when adding or subtracting measured quantities?

A. Use the smallest number of significant figures.
B. Use the fewest digits to the right of the decimal point.
C. Use the greatest number of decimal places.
D. Always round to one significant figure.

Answers

What do significant figures indicate about a measured quantity?

A. Its exact value
B. Its uncertainty
C. Its unit
D. Its conversion factor

Which rule is used when multiplying or dividing measured quantities?

A. Use the fewest decimal places.
B. Use the greatest number of significant figures.
C. Use the smallest number of significant figures.
D. Always use three significant figures.

Which rule is used when adding or subtracting measured quantities?

A. Use the smallest number of significant figures.
B. Use the fewest digits to the right of the decimal point.
C. Use the greatest number of decimal places.
D. Always round to one significant figure.

Pearson

Question 3

Calculate 12.5 ÷ 2.5 using the correct number of significant figures.

A. 5.00

B. 5.0

C. 5

D. 5.00

Solution:

Step 1: Divide the numbers

$12.5 \div 2.5 = 5$

Step 2: Count significant figures

- 12.5 has 3 significant figures.
- 2.5 has 2 significant figures.

For multiplication and division, the answer must have the same number of significant figures as the factor with the fewest significant figures.

The smallest number is 2 significant figures.
So we write:
$5 = 5.0$
because 5.0 has 2 significant figures.

Question 4

Calculate 12.35 + 4.2 using the correct number of significant figures.

A. 16.55

B. 16.6

C. 16.5

D. 17

Solution:

Step 1: Add the numbers

12.35 + 4.2 = 16.55

Step 2: Look at the decimal places

12.35 has 2 digits to the right of the decimal point.

4.2 has 1 digit to the right of the decimal point.

<u>For addition and subtraction, the answer must have the same number of decimal places as the number with the fewest decimal places.</u>

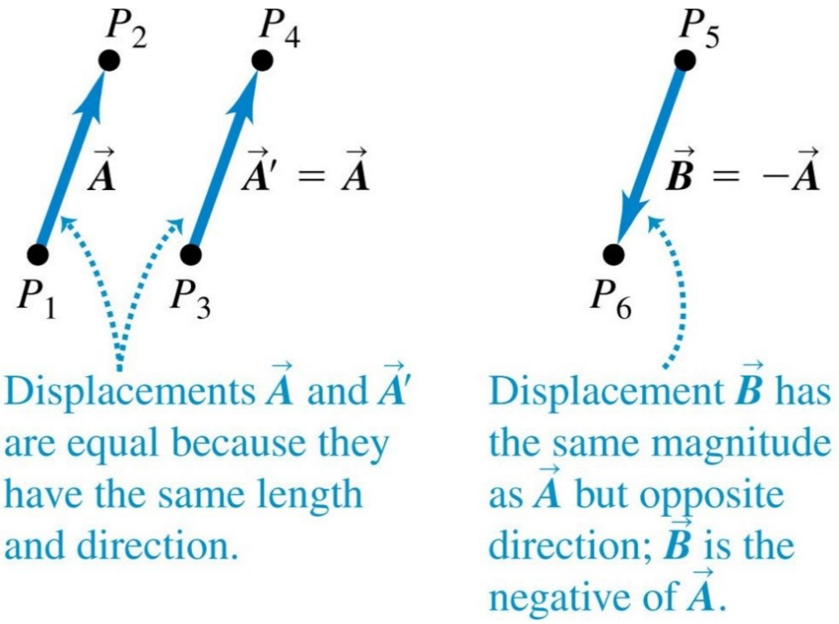
The fewest is 1 decimal place.
Therefore, round 16.55 to 1 decimal place:
$16.55 \rightarrow 16.6$

Vectors and Scalars

- A **scalar quantity** can be described by a **single number**.
- A **vector quantity** has both a **magnitude** and a **direction** in space.
- In this book, a vector quantity is represented in boldface italic type with an arrow over it: $\vec{\boldsymbol{A}}$.
- The magnitude of $\vec{\boldsymbol{A}}$ is written as A or $\left|\vec{\boldsymbol{A}}\right|$.

Drawing Vectors

- Draw a vector as a line with an arrowhead at its tip.
- The **length** of the line shows the vector's **magnitude**.
- The **direction** of the line shows the vector's **direction**.

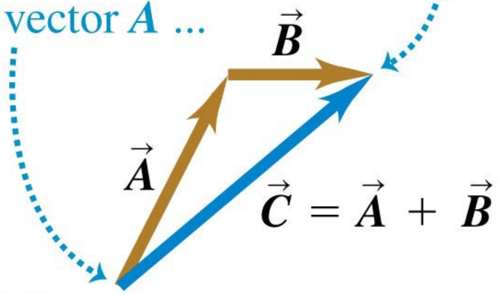


Displacements $\vec{A}$ and $\vec{A}'$ are equal because they have the same length and direction.

Displacement $\vec{B}$ has the same magnitude as $\vec{A}$ but opposite direction; $\vec{B}$ is the negative of $\vec{A}$.

Adding Two Vectors Graphically (1 of 3)

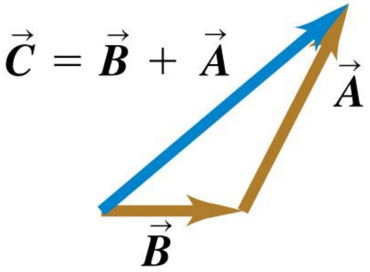
(a) We can add two vectors by placing them head to tail.



The vector sum $\vec{C}$ extends from the tail of vector $\vec{A}$ ...

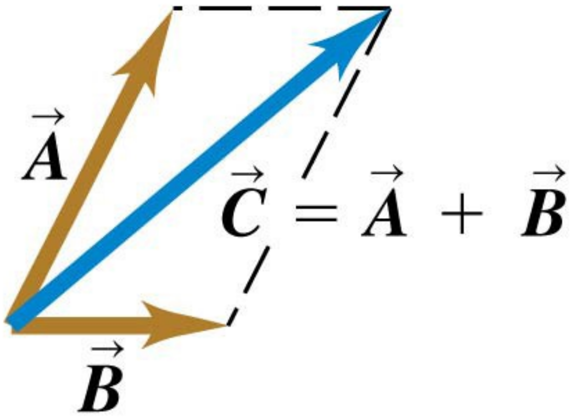
... to the head of vector $\vec{B}$.

(b) Adding them in reverse order gives the same result: $\vec{A} + \vec{B} = \vec{B} + \vec{A}$. The order doesn't matter in vector addition.



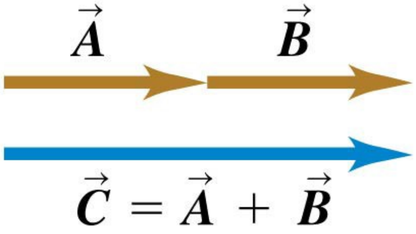
Adding Two Vectors Graphically (2 of 3)

(c) We can also add two vectors by placing them tail to tail and constructing a parallelogram.

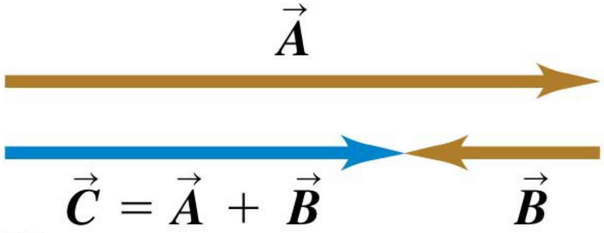


Adding Two Vectors Graphically (3 of 3)

(a) Only when vectors $\vec{A}$ and $\vec{B}$ are parallel does the magnitude of their vector sum $\vec{C}$ equal the sum of their magnitudes: $C = A + B$.

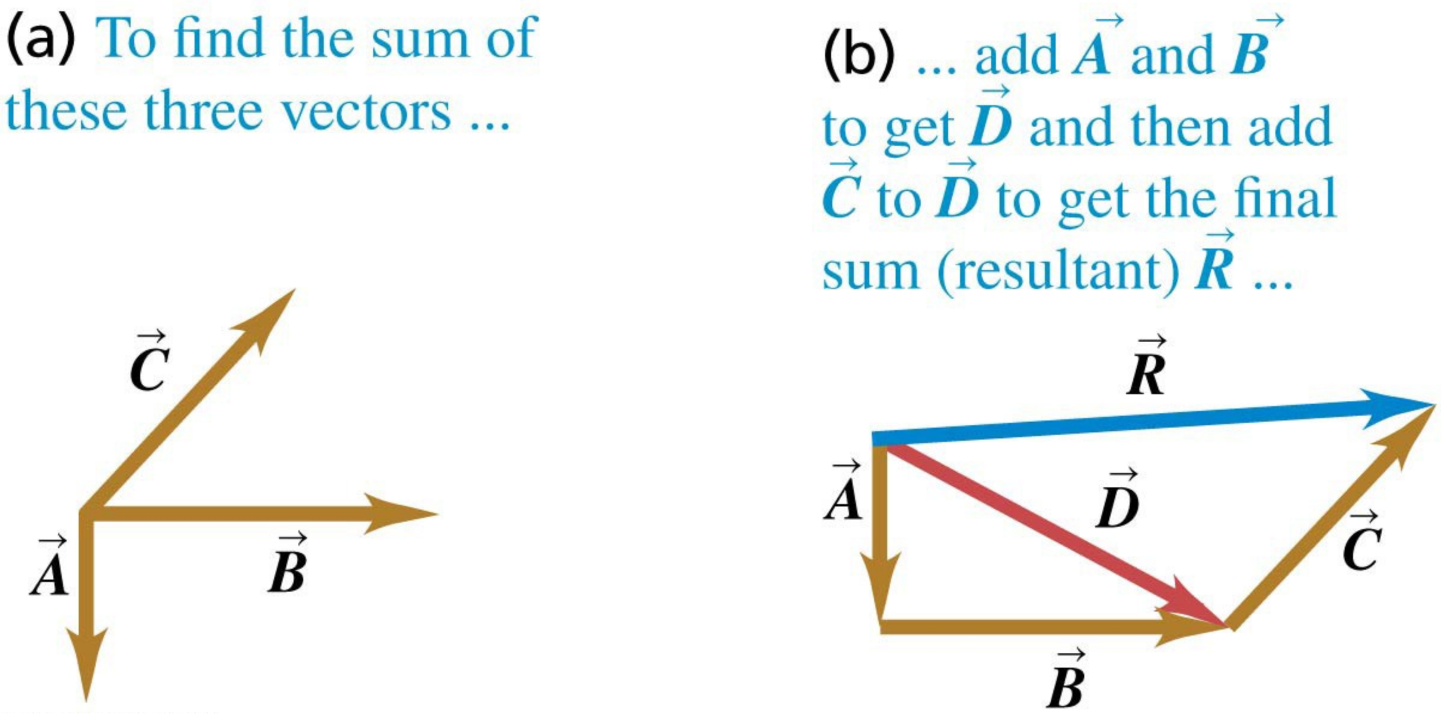


(b) When $\vec{A}$ and $\vec{B}$ are antiparallel, the magnitude of their vector sum $\vec{C}$ equals the *difference* of their magnitudes: $C = |A - B|$.



Adding More Than Two Vectors Graphically (1 of 3)

- To add several vectors, use the head-to-tail method.
- The vectors can be added in any order.

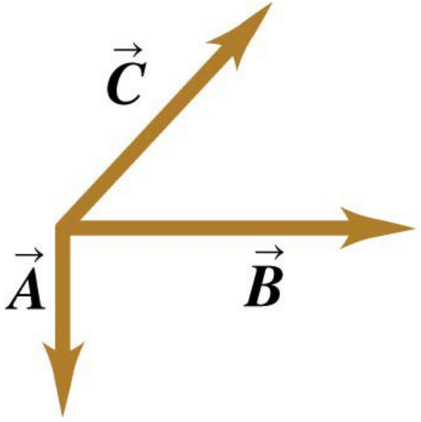


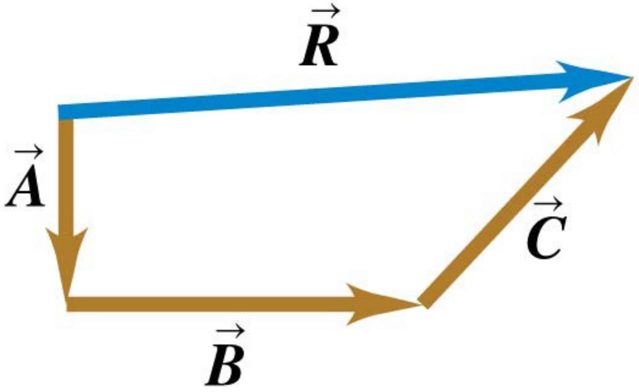
(a) To find the sum of these three vectors ...

(b) ... add $\vec{A}$ and $\vec{B}$ to get $\vec{D}$ and then add $\vec{C}$ to $\vec{D}$ to get the final sum (resultant) $\vec{R}$ ...

Adding More Than Two Vectors Graphically (2 of 3)

- To add several vectors, use the head-to-tail method.
- The vectors can be added in any order.

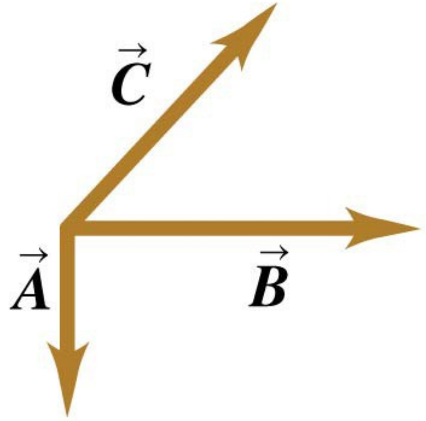

(a) To find the sum of these three vectors ...


(d) ... or add $\vec{A}$, $\vec{B}$, and $\vec{C}$ to get $\vec{R}$ directly ...

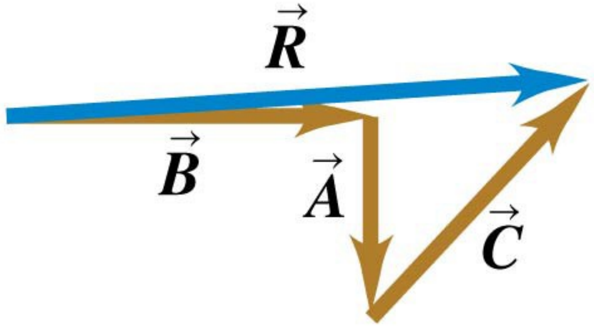
Adding More Than Two Vectors Graphically (3 of 3)

- To add several vectors, use the head-to-tail method.
- The vectors can be added in any order.

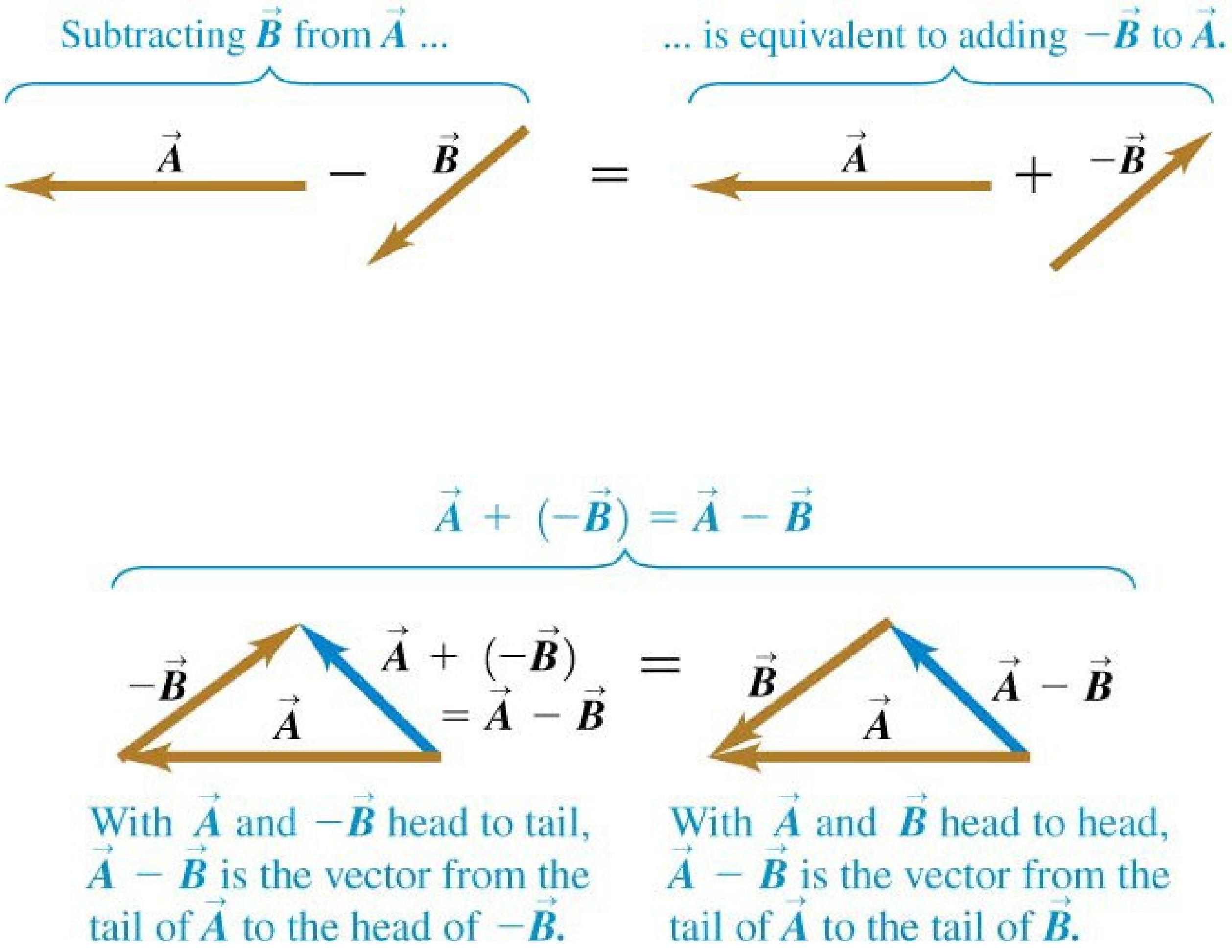
(a) To find the sum of these three vectors ...



(e) ... or add $\vec{A}$, $\vec{B}$, and $\vec{C}$ in any other order and still get $\vec{R}$.



Subtracting Vectors



Subtracting $\vec{B}$ from $\vec{A}$ ...

... is equivalent to adding $-\vec{B}$ to $\vec{A}$.

$\vec{A} + (-\vec{B}) = \vec{A} - \vec{B}$

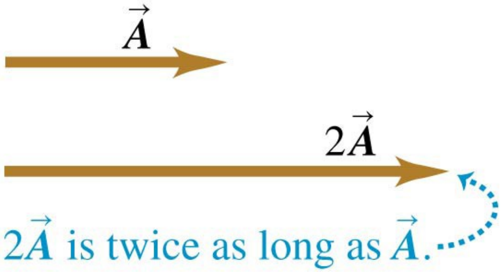
With $\vec{A}$ and $-\vec{B}$ head to tail, $\vec{A} - \vec{B}$ is the vector from the tail of $\vec{A}$ to the head of $-\vec{B}$.

With $\vec{A}$ and $\vec{B}$ head to head, $\vec{A} - \vec{B}$ is the vector from the tail of $\vec{A}$ to the tail of $\vec{B}$.

Multiplying a Vector by a Scalar

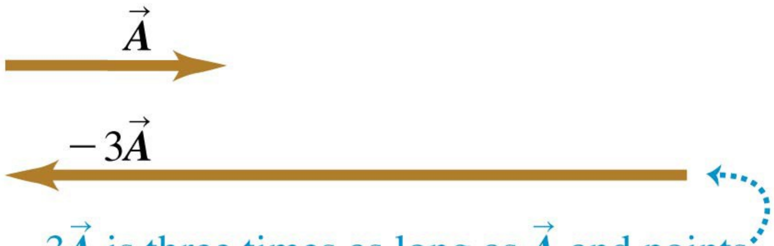
- If c is a scalar, the product $c\vec{A}$ has magnitude $|c|A$.
- The figure illustrates multiplication of a vector by
  a) a positive scalar and
  b) a negative scalar.

(a) Multiplying a vector by a positive scalar changes the magnitude (length) of the vector but not its direction.



$2\vec{A}$ is twice as long as $\vec{A}$.

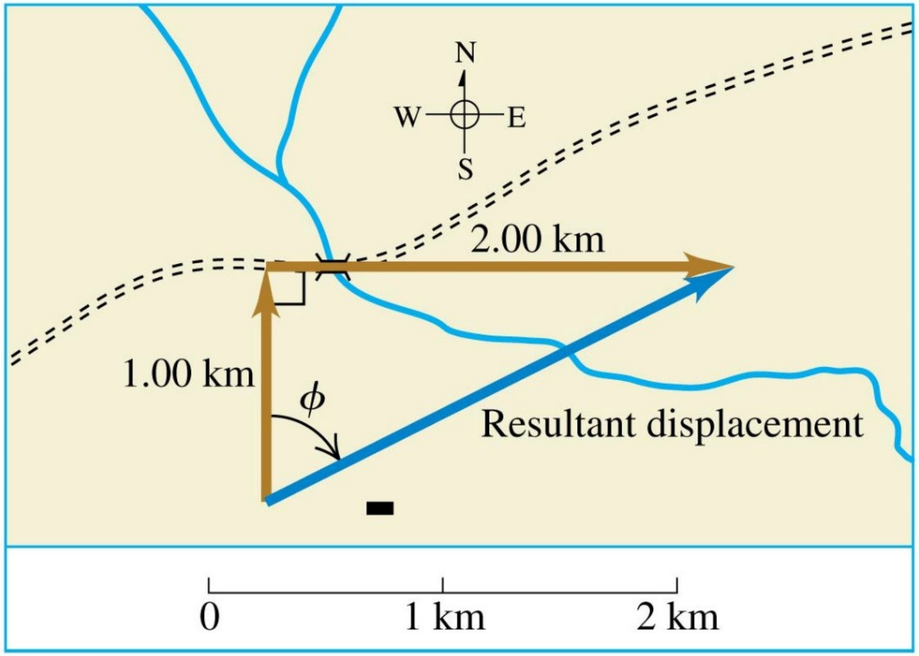
(b) Multiplying a vector by a negative scalar changes its magnitude and reverses its direction.



$-3\vec{A}$ is three times as long as $\vec{A}$ and points in the opposite direction.

Addition of Two Vectors at Right Angles

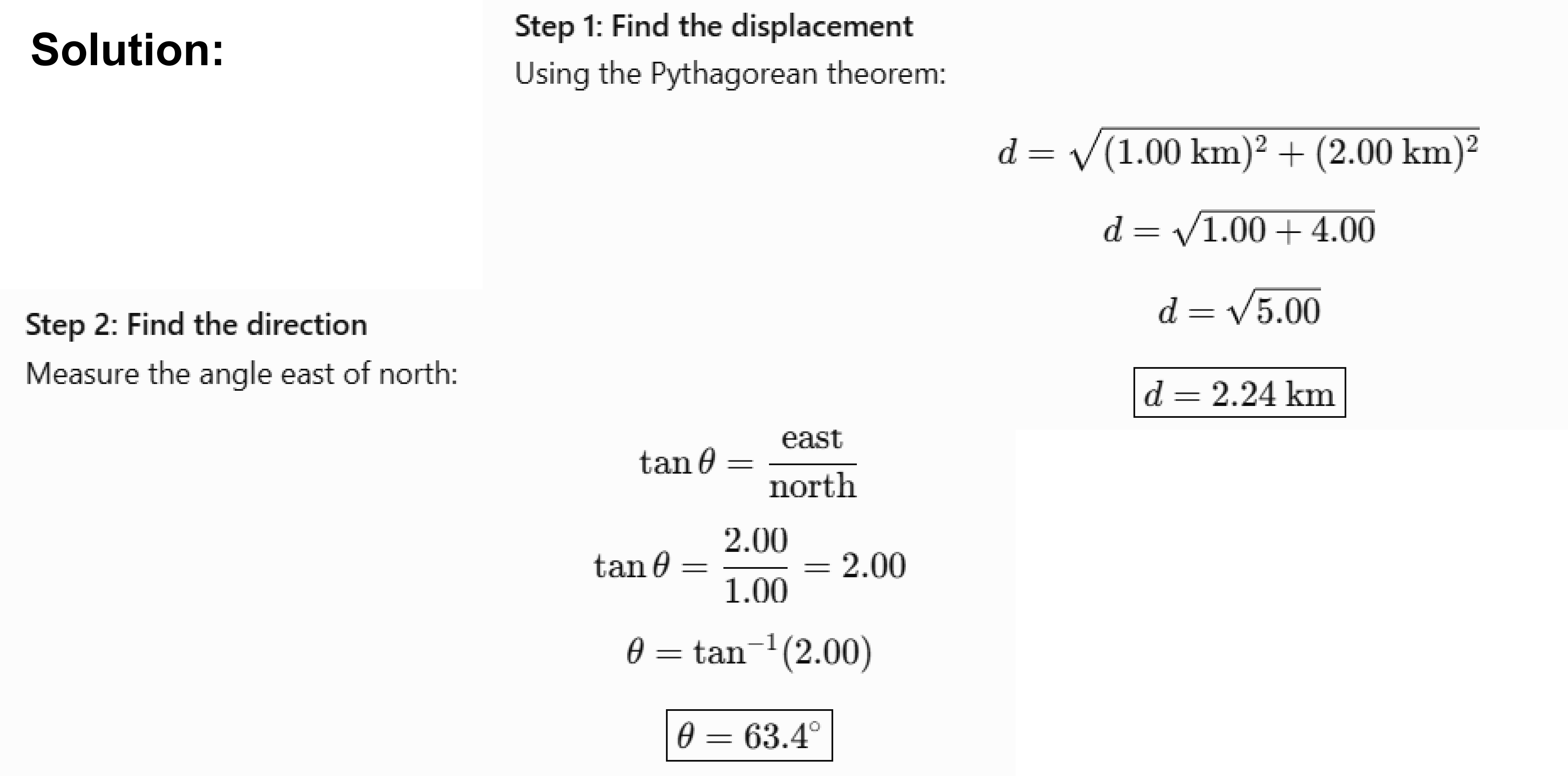
- To add two vectors that are at right angles, first add the vectors graphically.
- Then use trigonometry to find the magnitude and direction of the sum.
- In the figure, a cross-country skier ends up 2.24 km from her starting point, in a direction of 63.4 degrees east of north.



Question 5

A cross-country skier travels **1.00 km north** and then **2.00 km east** across a horizontal snowfield (Refer to previous slide image).
How far is she from her starting point, and in what direction?

Solution:



Step 1: Find the displacement

Using the Pythagorean theorem:

$$d = \sqrt{(1.00\ \mathrm{km})^2 + (2.00\ \mathrm{km})^2}$$

$$d = \sqrt{1.00 + 4.00}$$

$$d = \sqrt{5.00}$$

$$\boxed{d = 2.24\ \mathrm{km}}$$

Step 2: Find the direction

Measure the angle east of north:

$$\tan\theta = \frac{\mathrm{east}}{\mathrm{north}}$$

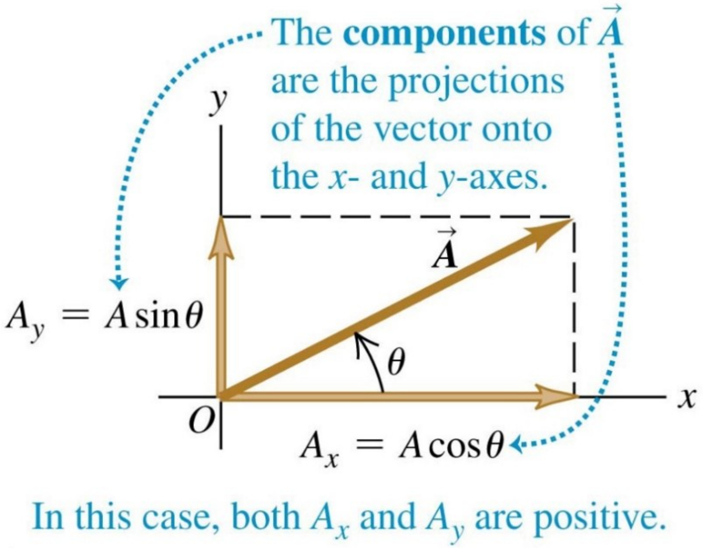
$$\tan\theta = \frac{2.00}{1.00} = 2.00$$

$$\theta = \tan^{-1}(2.00)$$

$$\boxed{\theta = 63.4^\circ}$$

Components of a Vector

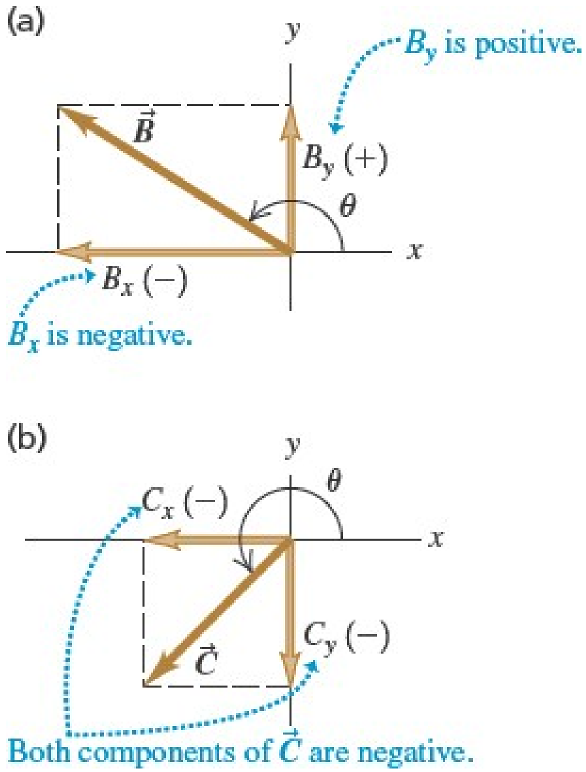
- Adding vectors graphically provides limited accuracy. Vector components provide a general method for adding vectors.
- Any vector can be represented by an *x*-component A_x and a *y*-component A_y.



In this case, both A_x and A_y are positive.

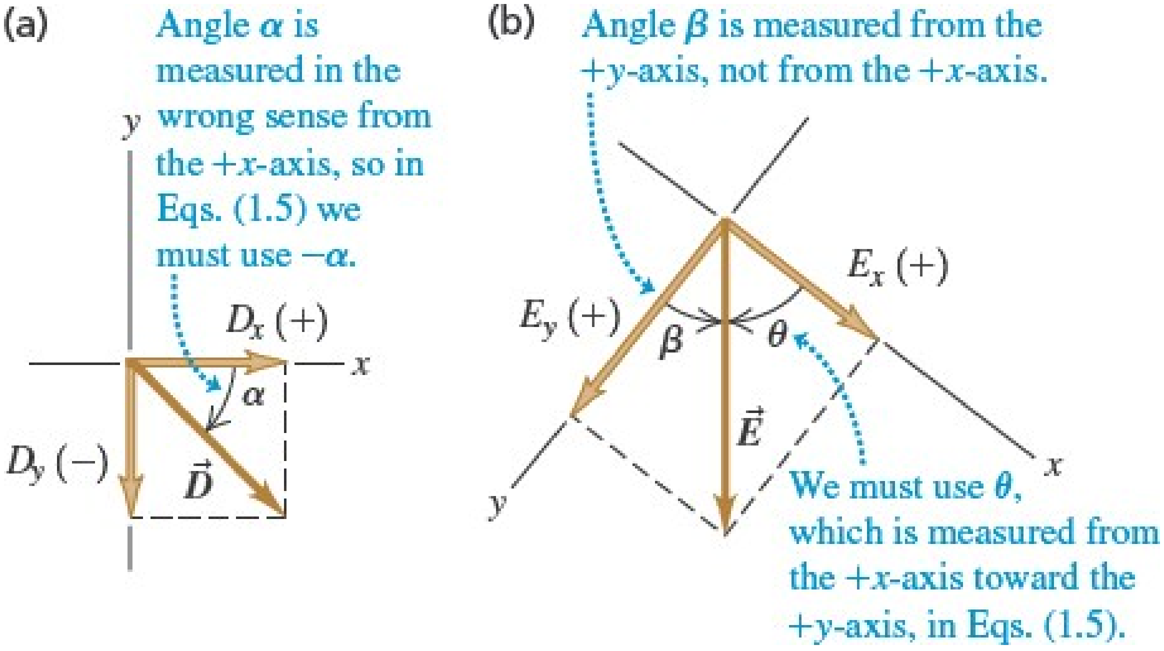
Positive and Negative Components

- The components of a vector may be positive or negative numbers, as shown in the figures.



Finding Components

- We can calculate the components of a vector from its magnitude and direction.



Angle α is measured in the wrong sense from the +x-axis, so in Eqs. (1.5) we must use −α.

Angle β is measured from the +y-axis, not from the +x-axis.

We must use θ, which is measured from the +x-axis toward the +y-axis, in Eqs. (1.5).

- Video Tutor Solution: Example 1.6

Question 6

For each vector below, determine its **x- and y-components**.

(a) Vector D has a magnitude of 3.00 m and makes an angle of α=*45.0°*.

(b) Vector E has a magnitude of 4.50 m and makes an angle of β= *37.0°*.

Use the diagrams provided, in previous slide, to determine the appropriate signs of the x- and y-components.

Solution:

$$D_x = (3.00)\cos 45^\circ = +2.12\text{ m}$$

$$D_y = -(3.00)\sin 45^\circ = -2.12\text{ m}$$

$$E_x = (4.50)\sin 37^\circ = +2.71\text{ m}$$

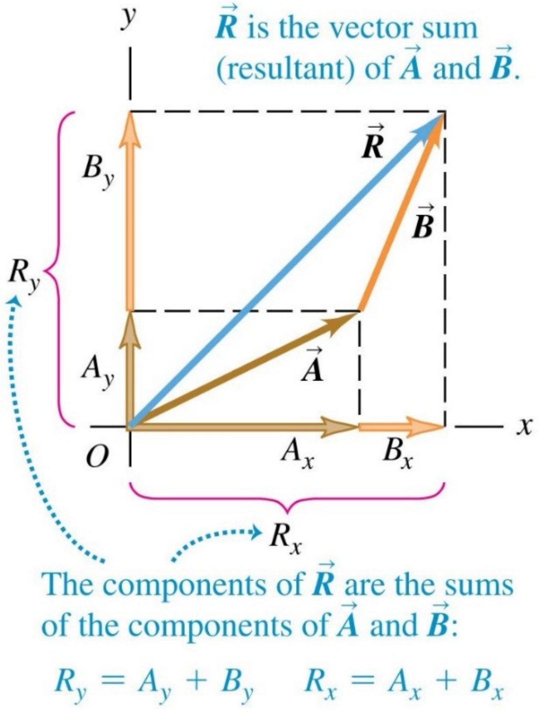
$$E_y = (4.50)\cos 37^\circ = +3.59\text{ m}$$

Pearson

Calculations Using Components

- We can use the components of a vector to find its magnitude and direction: $A = \sqrt{A_x^{\,2} + A_y^{\,2}}$ and $\tan\theta = \dfrac{A_y}{A_x}$
- We can use the components of a set of vectors to find the components of their sum:

$$R_x = A_x + B_x + C_x + \ldots,\ R_y = A_y + B_y + C_y + \ldots$$


$\vec{R}$ is the vector sum (resultant) of $\vec{A}$ and $\vec{B}$.

The components of $\vec{R}$ are the sums of the components of $\vec{A}$ and $\vec{B}$:

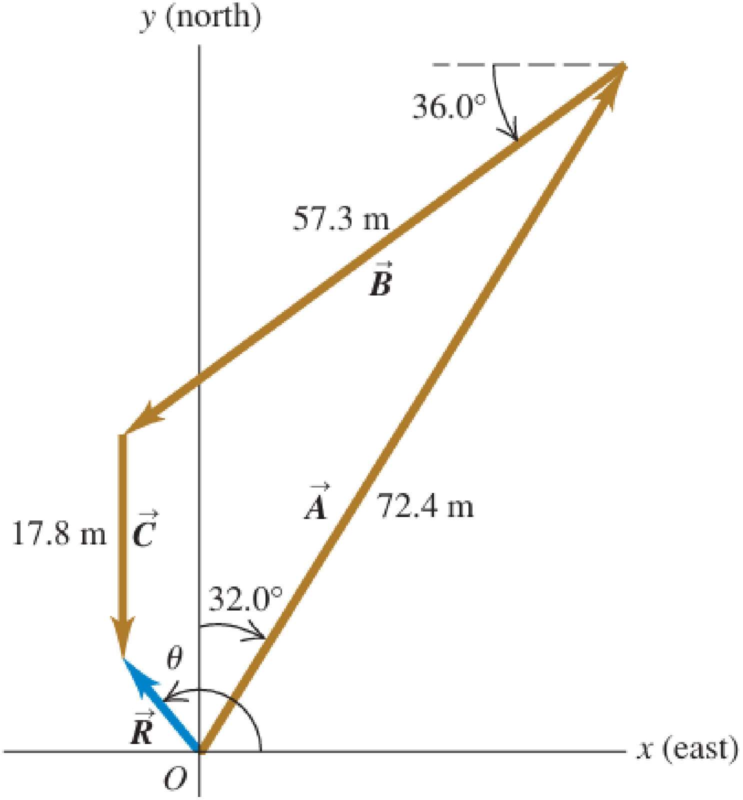
$R_y = A_y + B_y \qquad R_x = A_x + B_x$

Question 7

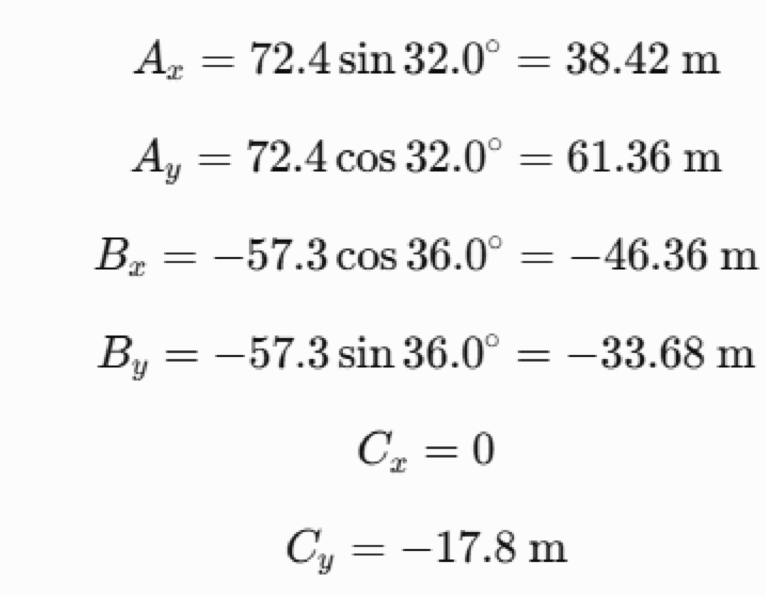
Three hikers are searching for a hidden treasure in the center of a large, flat field. Each hiker is given a compass, a measuring tape, and a calculator. They are told that the treasure can be found by following these three displacements, in any order:

- 72.4 m, 32.0◦ east of north
- 57.3 m, 36.0◦ south of west
- 17.8 m due south

One hiker decides to calculate the **resultant displacement** before starting to walk. What displacement should she follow to reach the treasure directly?



Solution: Draw the vectors and find their components



$$A_x = 72.4 \sin 32.0^\circ = 38.42 \text{ m}$$

$$A_y = 72.4 \cos 32.0^\circ = 61.36 \text{ m}$$

$$B_x = -57.3 \cos 36.0^\circ = -46.36 \text{ m}$$

$$B_y = -57.3 \sin 36.0^\circ = -33.68 \text{ m}$$

$$C_x = 0$$

$$C_y = -17.8 \text{ m}$$

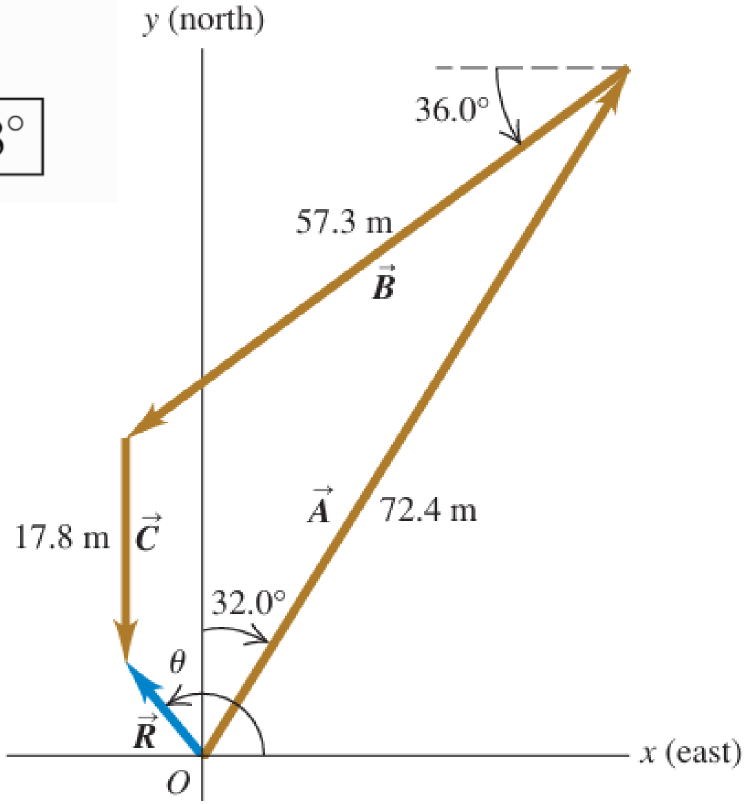
$$R_x = 38.42 - 46.36 = -7.94 \text{ m}$$

$$R_y = 61.36 - 33.68 - 17.8 = 9.88 \text{ m}$$

$$R = \sqrt{(-7.94)^2 + (9.88)^2} = 12.68 \text{ m}$$

$$\theta = \tan^{-1}\left(\frac{7.94}{9.88}\right) = 38.8^\circ$$

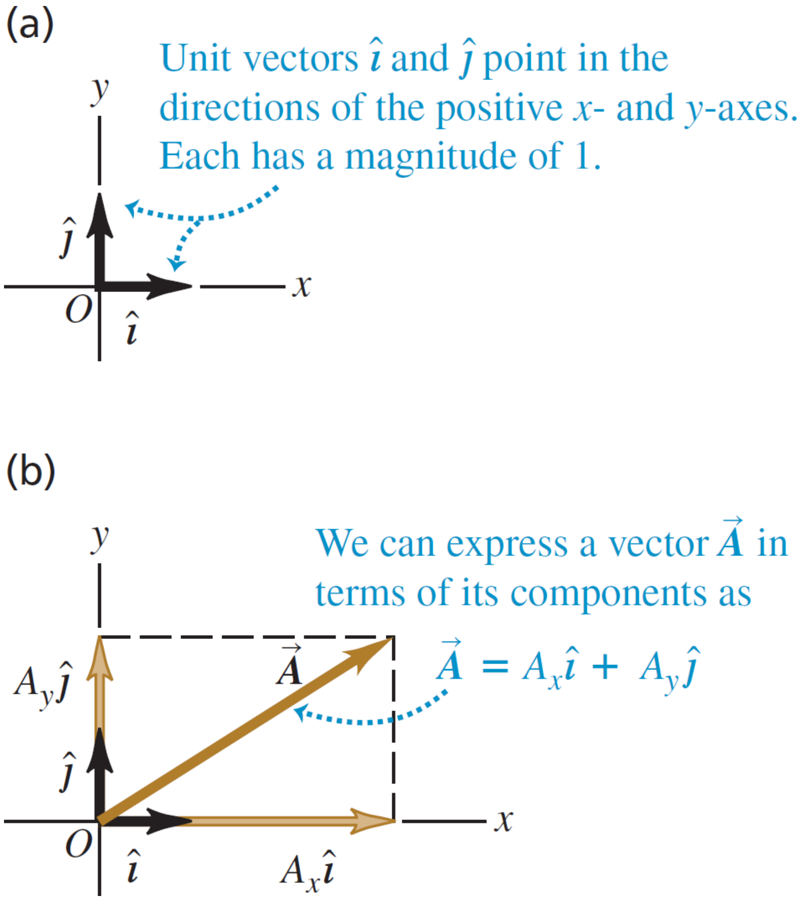
$$\boxed{R = 12.7 \text{ m west of north at } 38.8^\circ}$$



Unit Vectors

- A **unit vector** has a magnitude of 1 with no units.
- The unit vector $\hat{\boldsymbol{i}}$ points in the $+x$-direction, $\hat{\boldsymbol{j}}$ points in the $+y$-direction, and $\hat{\boldsymbol{k}}$ points in the $+z$-direction.
- Any vector can be expressed in terms of its components as

$$\vec{\boldsymbol{A}} = A_x\hat{\boldsymbol{i}} + A_y\hat{\boldsymbol{j}} + A_z\hat{\boldsymbol{k}}.$$

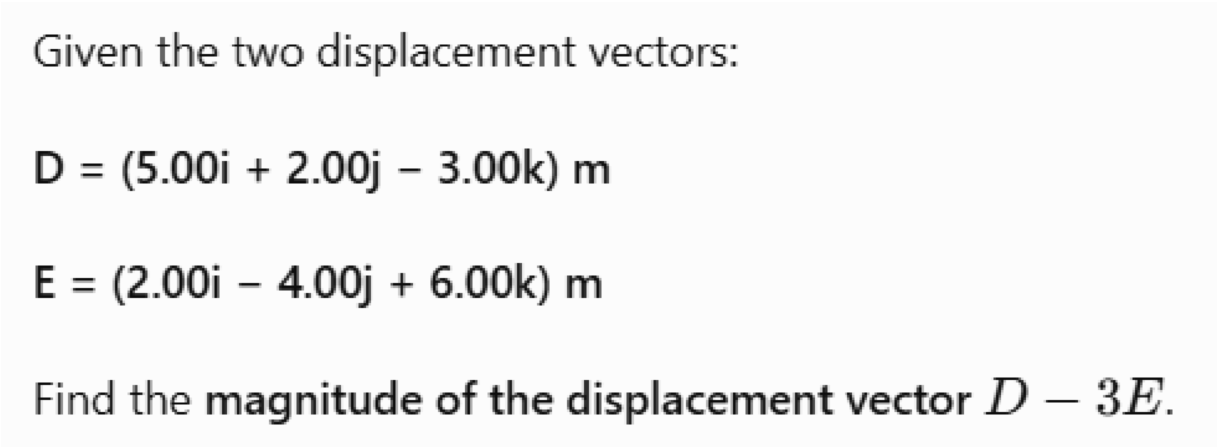


Unit vectors $\hat{\imath}$ and $\hat{j}$ point in the directions of the positive x- and y-axes. Each has a magnitude of 1.

We can express a vector $\vec{A}$ in terms of its components as

$$\vec{A} = A_x\hat{\imath} + A_y\hat{j}$$

Question 8



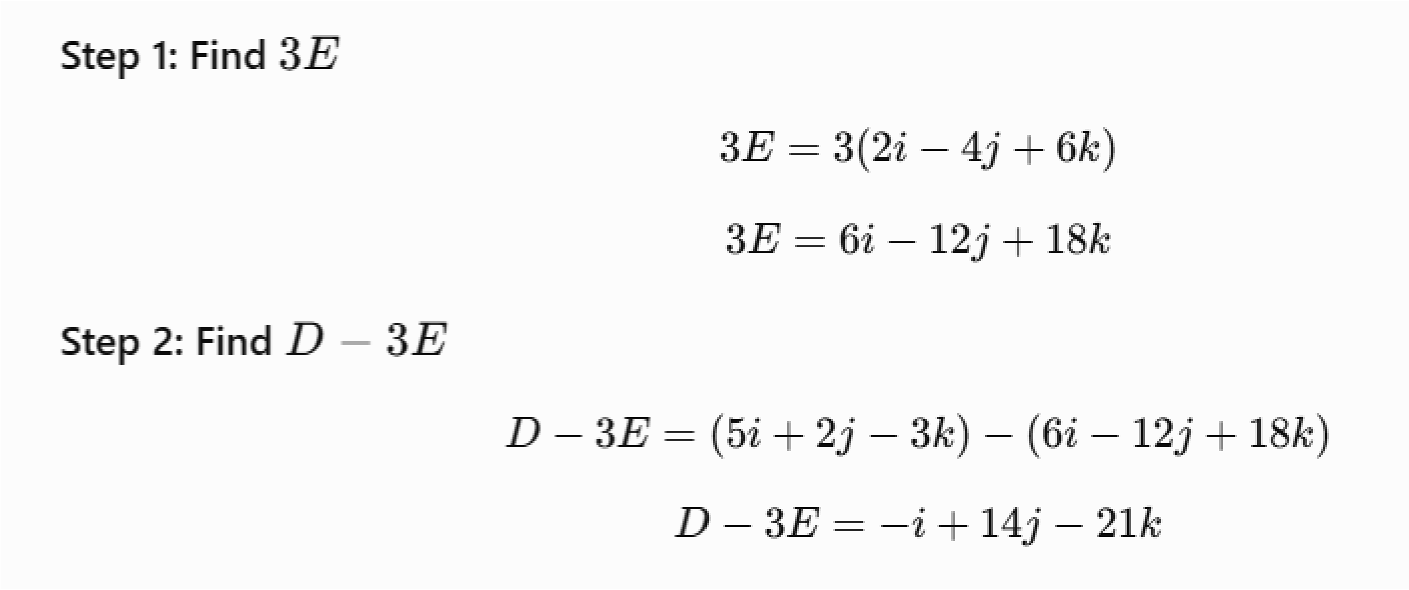
Given the two displacement vectors:

D = (5.00i + 2.00j – 3.00k) m

E = (2.00i – 4.00j + 6.00k) m

Find the **magnitude of the displacement vector** $D - 3E$.

Solution:



Step 1: Find $3E$

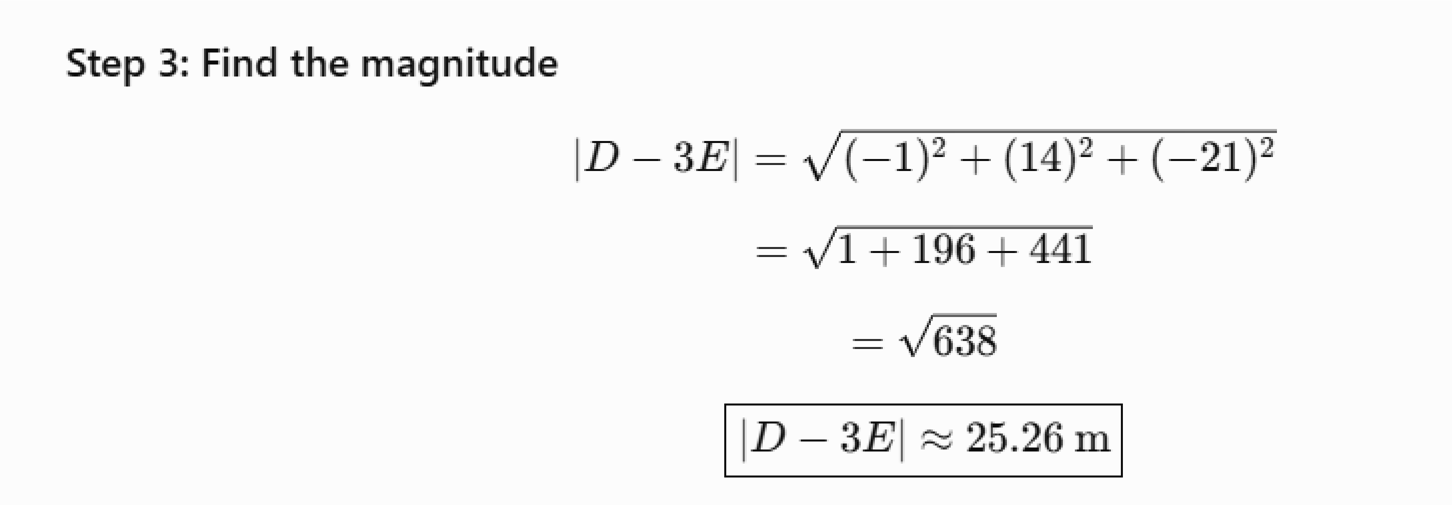
$$3E = 3(2i - 4j + 6k)$$

$$3E = 6i - 12j + 18k$$

Step 2: Find $D - 3E$

$$D - 3E = (5i + 2j - 3k) - (6i - 12j + 18k)$$

$$D - 3E = -i + 14j - 21k$$



Step 3: Find the magnitude

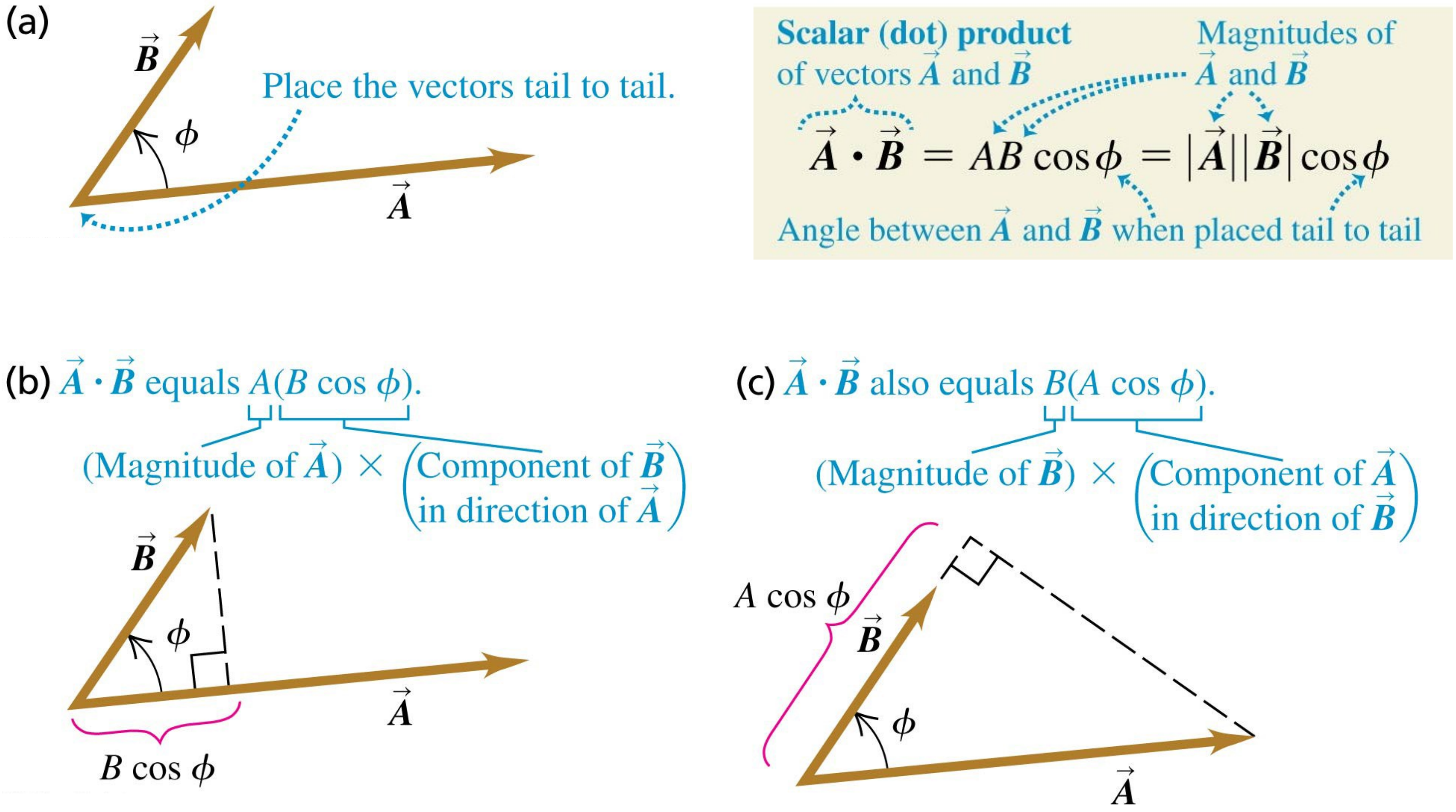
$$|D-3E| = \sqrt{(-1)^2+(14)^2+(-21)^2}$$

$$= \sqrt{1+196+441}$$

$$= \sqrt{638}$$

$$\boxed{|D-3E| \approx 25.26\ \mathrm{m}}$$

The Scalar Product (1 of 2)



Scalar (dot) product of vectors $\vec{A}$ and $\vec{B}$

Magnitudes of $\vec{A}$ and $\vec{B}$

$$\vec{A}\cdot\vec{B} = AB\cos\phi = |\vec{A}||\vec{B}|\cos\phi$$

Angle between $\vec{A}$ and $\vec{B}$ when placed tail to tail

(b) $\vec{A}\cdot\vec{B}$ equals $A(B\cos\phi)$.

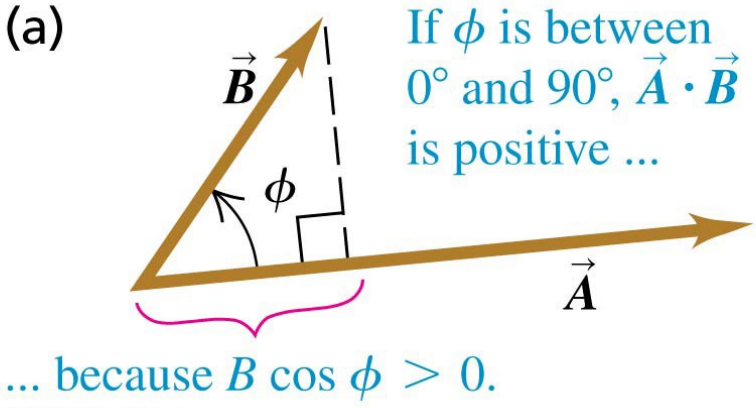
(Magnitude of $\vec{A}$) × (Component of $\vec{B}$ in direction of $\vec{A}$)

(c) $\vec{A}\cdot\vec{B}$ also equals $B(A\cos\phi)$.

(Magnitude of $\vec{B}$) × (Component of $\vec{A}$ in direction of $\vec{B}$)

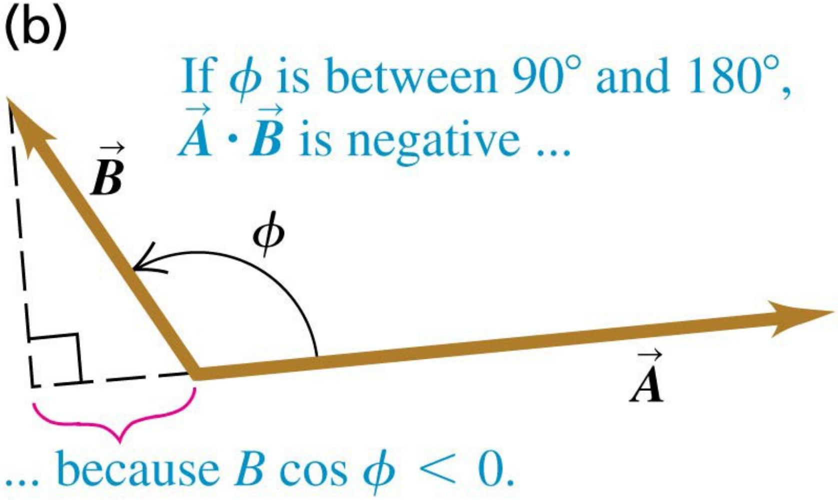
The Scalar Product (2 of 2)

- The scalar product can be positive, negative, or zero, depending on the angle between $\vec{A}$ and $\vec{B}$.



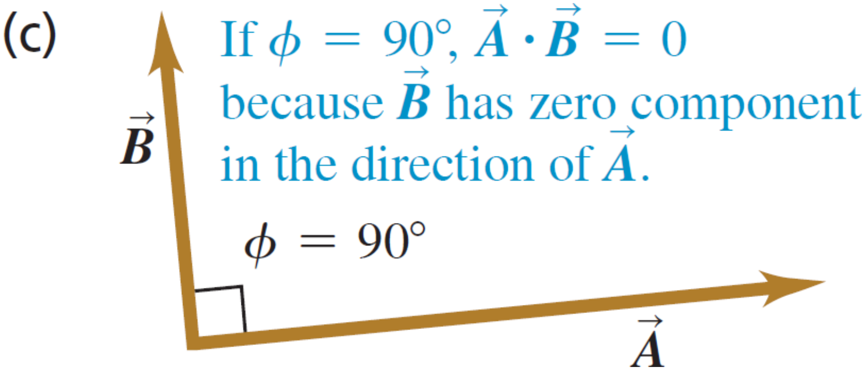
If ϕ is between 0° and 90°, $\vec{A} \cdot \vec{B}$ is positive ...

... because $B \cos \phi > 0$.



If ϕ is between 90° and 180°, $\vec{A} \cdot \vec{B}$ is negative ...

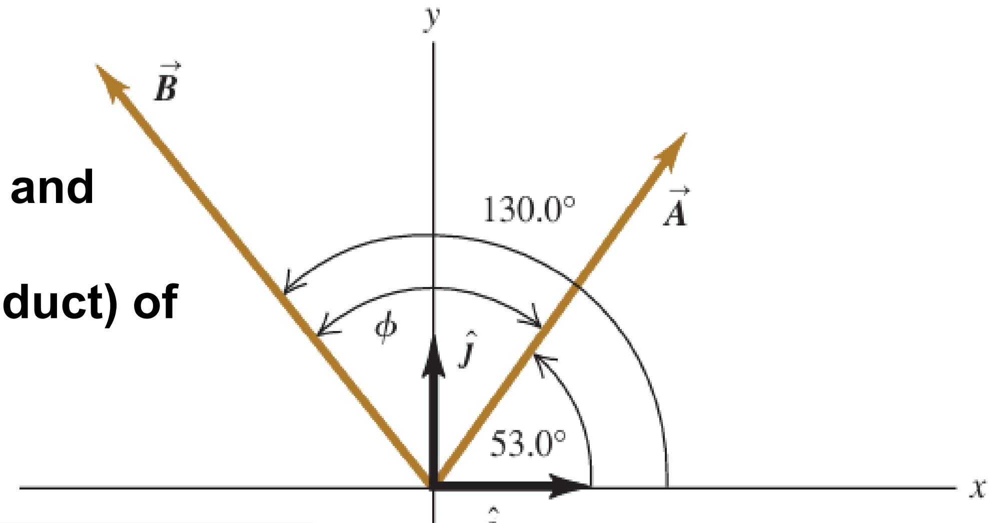
... because $B \cos \phi < 0$.



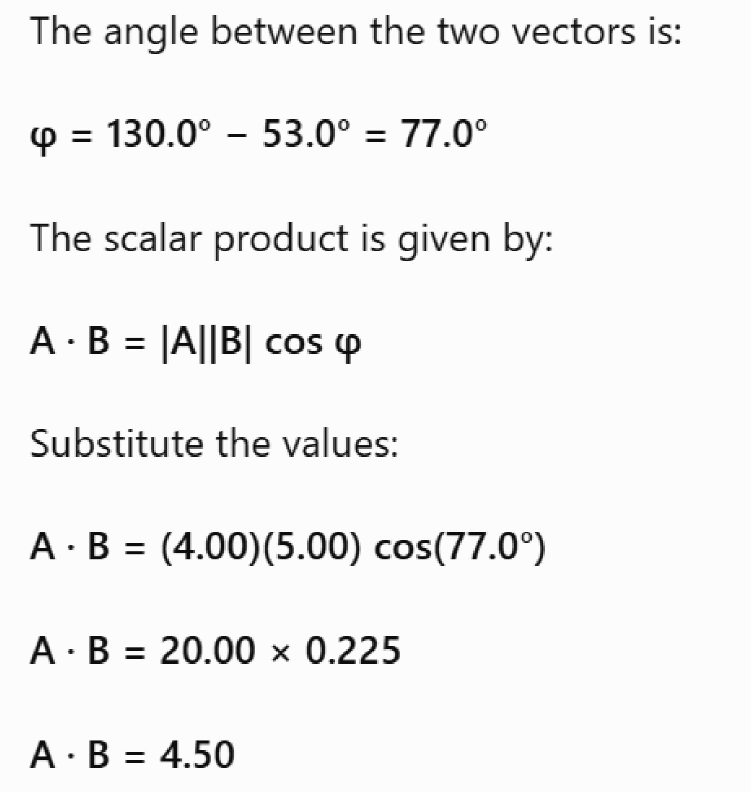
If $\phi = 90°$, $\vec{A} \cdot \vec{B} = 0$ because $\vec{B}$ has zero component in the direction of $\vec{A}$.

Question 9

Two vectors **A** and **B** are shown.
The magnitude of vector **A** is **4.00 and** vector **B** is **5.00**.
Find the scalar product (dot product) of the two vectors, A · B.



Solution:



The angle between the two vectors is:

φ = 130.0° − 53.0° = 77.0°

The scalar product is given by:

A · B = |A||B| cos φ

Substitute the values:

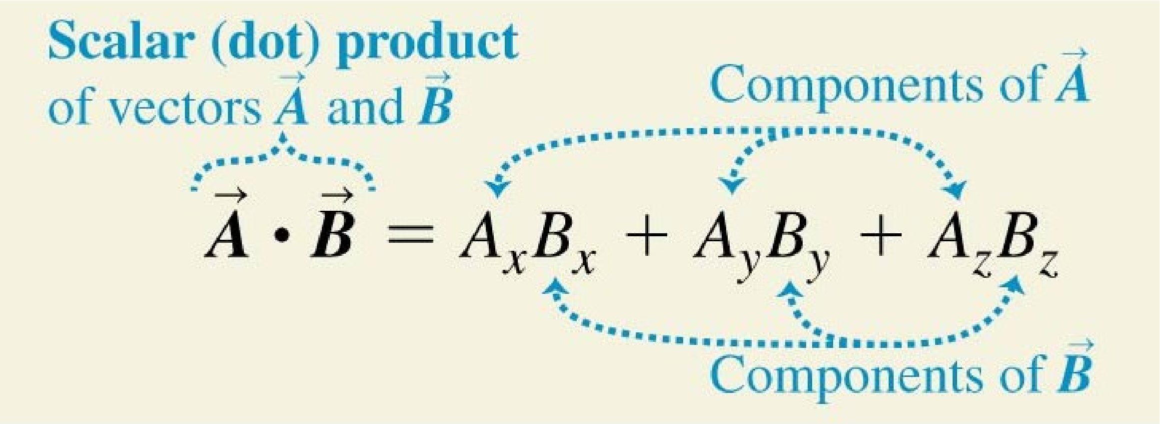
A · B = (4.00)(5.00) cos(77.0°)

A · B = 20.00 × 0.225

A · B = 4.50

Calculating a Scalar Product Using Components

- In terms of components:



Scalar (dot) product of vectors $\vec{A}$ and $\vec{B}$

Components of $\vec{A}$

$$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$$

Components of $\vec{B}$

- The scalar product of two vectors is the sum of the products of their respective components.

Question 10

Find the angle between the vectors

$$\vec{A} = 2.00\hat{\imath} + 3.00\hat{\jmath} + 1.00\hat{k} \quad \text{and}$$
$$\vec{B} = -4.00\hat{\imath} + 2.00\hat{\jmath} - 1.00\hat{k}$$

Solution:

Use the formula:

$$\cos\theta = \frac{\vec{A}\cdot\vec{B}}{|\vec{A}||\vec{B}|}$$

1. Dot product:

$$\vec{A}\cdot\vec{B} = (2)(-4) + (3)(2) + (1)(-1) = -8 + 6 - 1 = -3$$

2. Magnitudes:

$$|\vec{A}| = \sqrt{2^2 + 3^2 + 1^2} = \sqrt{14}$$

$$|\vec{B}| = \sqrt{(-4)^2 + 2^2 + (-1)^2} = \sqrt{21}$$

3. Find angle:

$$\cos\theta = \frac{-3}{\sqrt{14}\sqrt{21}}$$

$$\theta \approx 100^\circ$$

The Vector Product

If the vector product (“cross product”) of two vectors is $\vec{C} = \vec{A} \times \vec{B}$ then:

Magnitude of **vector (cross) product** of vectors $\vec{B}$ and $\vec{A}$

$$C = AB \sin\phi$$

Magnitudes of $\vec{A}$ and $\vec{B}$

Angle between $\vec{A}$ and $\vec{B}$ when placed tail to tail

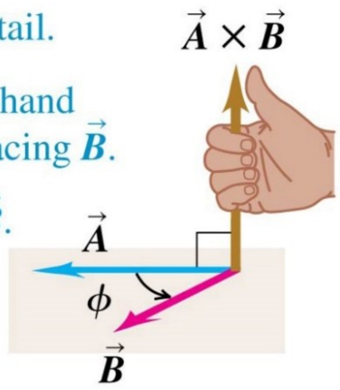
The direction of the vector product can be found using the right-hand rule:

① Place $\vec{A}$ and $\vec{B}$ tail to tail.

② Point fingers of right hand along $\vec{A}$, with palm facing $\vec{B}$.

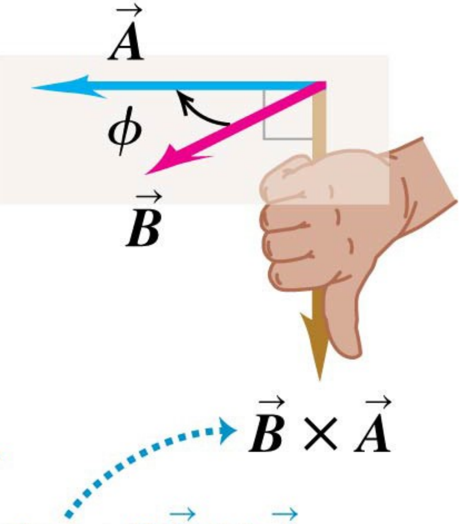
③ Curl fingers toward $\vec{B}$.

④ Thumb points in direction of $\vec{A} \times \vec{B}$.



The Vector Product is Anticommutative

$$\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$$



① Place $\vec{B}$ and $\vec{A}$ tail to tail.

② Point fingers of right hand along $\vec{B}$, with palm facing $\vec{A}$.

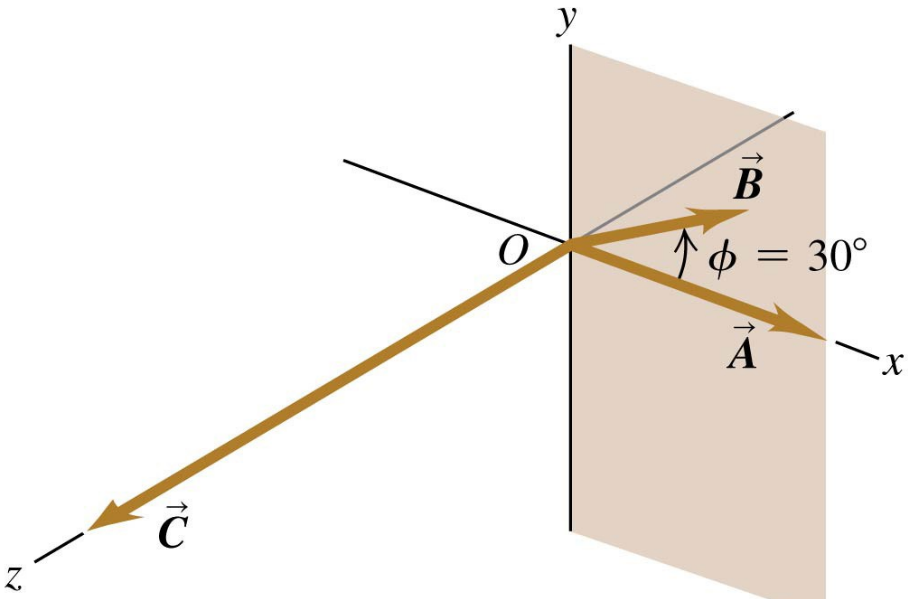
③ Curl fingers toward $\vec{A}$.

④ Thumb points in direction of $\vec{B} \times \vec{A}$.

⑤ $\vec{B} \times \vec{A}$ has same magnitude as $\vec{A} \times \vec{B}$ but points in opposite direction.

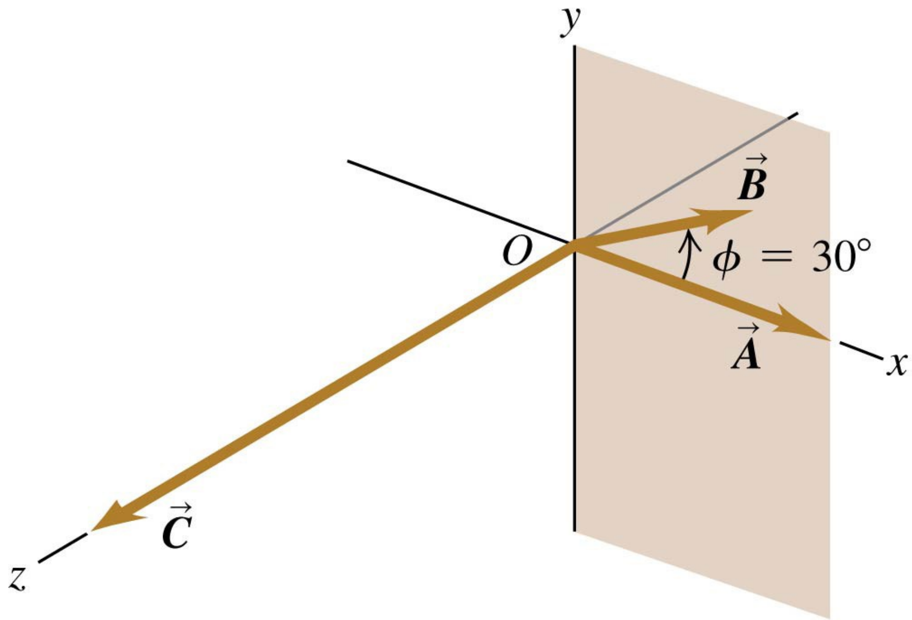
Calculating the Vector Product

- Use $AB\sin\phi$ to find the magnitude and the right-hand rule to find the direction.
- Refer to Example 1.11.



- Video Tutor Solution: Example 1.11

Question 11 Vector $\vec{A}$ has a magnitude of 6 units and points along the $+x$-axis. Vector $\vec{B}$ has a magnitude of 4 units and lies in the xy-plane, making an angle of $30°$ with the $+x$-axis:



Solution

$$\vec{A} = (6, 0, 0), \qquad \vec{B} = (2\sqrt{3}, 2, 0)$$

$$C_x = A_y B_z - A_z B_y = 0$$

$$C_y = A_z B_x - A_x B_z = 0$$

$$C_z = A_x B_y - A_y B_x = 12$$

$$\vec{C} = (C_x, C_y, C_z) = (0, 0, 12)$$

$$\vec{C} = 12\hat{k}.$$